Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.1 Different Theories of Earth Pressure]

Objectives

In this section you will learn the following

- Introduction
- Different Theories of Earth Pressure
- Lateral Earth Pressure For At Rest Condition
- Movement of the Wall
- Different Movements

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.1 Different Theories of Earth Pressure]

6 Introduction

A soil mass is stable when the slope of the surface of the soil mass is flatter than the safe slope. At some locations, due to limitation of space, it s not possible to provide flat slope & the soil is to be retained at a slope steeper than the safe one. In such cases, a retaining structure is required to provide lateral support to the soil mass.

A retaining structure is a permanent or temporary structure which is used for providing lateral support to the soil mass or other materials. Some of the examples of retaining structures used in soil & foundation engineering are: Retaining wall, Sheet piles, Anchored Bulkheads, Sheeting & Basement wall, etc. Generally, the soil masses are vertical or nearly vertical behind the retaining structure. Thus, a retaining structure maintains the soil at different elevations on its either side. In the absence of a retaining structure, the soil on the higher side would have a tendency to slide and may not remain stable.

The design of the retaining structure requires the determination of the magnitude & line of action of the lateral earth pressure. The magnitude of the lateral earth pressure depends upon a number of factors, such as the mode of movement of the wall, the flexibility of the wall, the properties of the soil, the drainage conditions. For convenience, the retaining wall is assumed to be rigid & the soil structure interaction effect is neglected which arises due to the flexibility of the wall.

The lateral earth pressure is usually computed using the classical theories proposed by Coulomb (1773) & Rankine (1857). The general wedge theory proposed by Terzaghi (1943) is more general and is an improvement over the earlier theories. The general equations developed for both theories are based on the fundamental assumptions that

the retained soil is **cohesionless** (no clay component), **homogeneous** (not a varying mixture of materials), **isotropic** (similar stress-strain properties in all directions or in practical terms, not reinforced), **semi-infinite** (wall is very long and soil goes back a long distance without bends or other boundary conditions), and **well drained** to avoid consideration of pore pressures.

The pressure or force exerted by soil on any boundary is called the earth pressure. When the earth pressure acts on the side (back or face) of a retaining wall, it is known as the Lateral earth pressure. The magnitude of the lateral earth pressure depends upon the movement of the retaining wall relative to the backfill & upon the nature of the soil.

Module 2 :Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.1 Different Theories of Earth Pressure]

6.1 Different Theories of Earth Pressure

There are two classical theories of earth pressure. They are

- Coulomb's earth pressure theory.
- Rankine's earth pressure theory.

Coulomb published the first rigorous analysis of lateral earth pressure problem in 1776. Rankine proposed a different approach to the problem in 1857. These theories propose to estimate the magnitudes of two pressures called *Active earth pressure* and *Passive earth pressure* . All the theories proposed by Coulomb have been discussed in the later part of the chapter.

Assumptions

Most of the theories of earth pressure are based on the following assumptions: the backfill of the wall is isotropic and homogenous; the deformation of the backfill occurs exclusively parallel to the vertical plane at right angles at the back of the wall, and the neutral stresses in the backfill material are negligible.

- The soil mass is semi infinite, homogeneous, dry & cohesion less.
- The ground surface is plane which may be horizontal or inclined.
- The back of the wall is smooth & vertical. In other words, there are no shearing stresses or frictional stresses between the wall & the soil. The stress relationship for any element adjacent to the wall is the same as for any other element farther away from the wall.
- The soil mass is in a state of plastic equilibrium i.e. at the verge of failure.

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.1 Different Theories of Earth Pressure]

The earth pressures are defined as follows:

Consider a retaining wall with a plane vertical face, as shown in Fig. 2.1, which is backfilled with cohesionless soil. If the wall does not move even after filling the materials, the pressure exerted on the wall is known as pressure for the *at rest condition* of the wall. If suppose the wall gradually rotates about the point A and moves away from the backfill, the unit pressure on the wall gradually gets reduced and after a particular displacement of the wall at the top, the pressure reaches a constant value. This pressure is the minimum possible. The pressure is termed as the active earth pressure since the weight of the backfill is responsible for the movement of the wall. If the wall is smooth, the resultant pressure acts normal to the face of the wall. If the wall is rough, the resultant pressure acts at an angle of δ to the normal to the face. This angle δ is known as *the angle of wall friction* . As the wall moves away from the backfill, the soil also tends to move forward. When the wall movement is sufficient, a soil mass of weight W ruptures along a surface AC'C shown in fig. This surface is slightly curved. If the surface is assumed to be plane surface AC, analysis would indicate that this surface would make an angle of $45^0+\phi/2$ with the horizontal.

If the wall is now rotated about A towards the backfill, the actual failure plane AC'C is also a curved surface. However, if the failure surface is approximated to a plane AC, this makes an angle $45^0+\phi/2$ with the horizontal and the pressure on the wall increases from the value of at rest condition to a maximum possible value. The maximum pressure p_p that is developed is termed as passive earth pressure. The pressure is called passive earth pressure because the weight of the backfill opposes the movement of the wall. It also makes an angle of δ with the normal if the wall is rough.

The gradual increase or decrease of the pressure of the wall with the movement of the wall from the “at rest condition” may be depicted as shown in Fig.2.2. The movement Δp required to develop passive state is considerably larger than the Δ_a required for active case.

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.1 Different Theories of Earth Pressure]

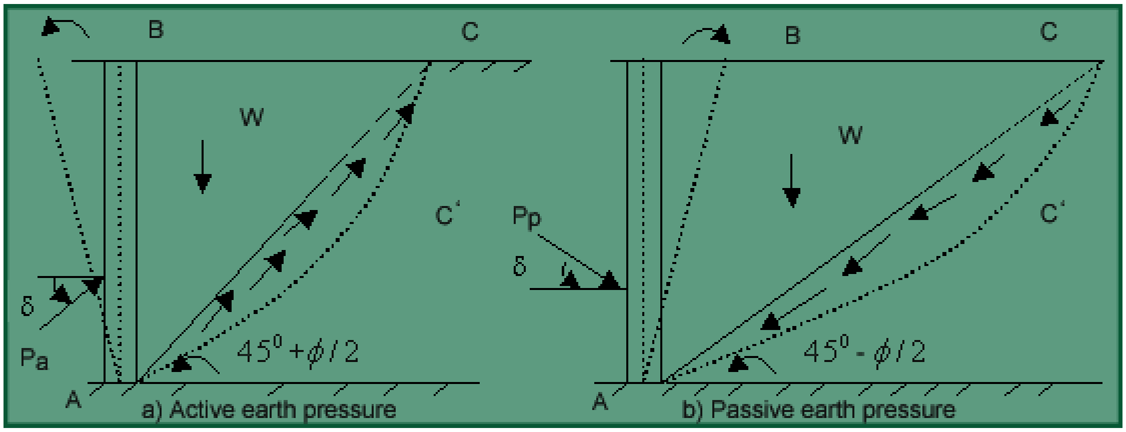


Fig 2.1. Wall movement for the development of active and passive earth

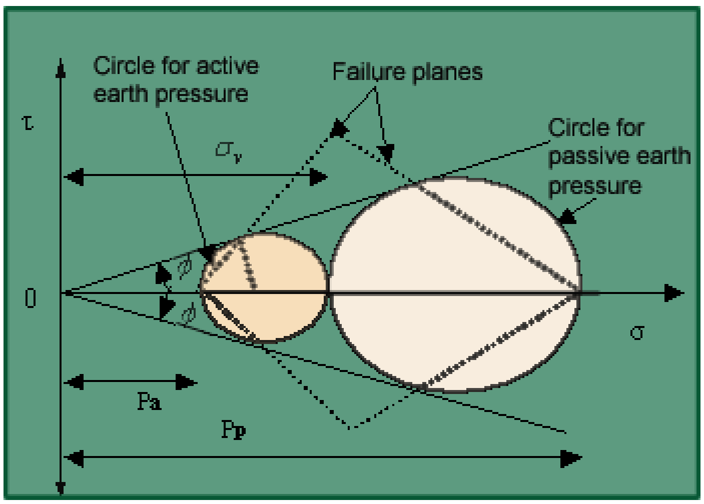


Fig.2.2. Mohr stress diagram

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.1 Different Theories of Earth Pressure]

1. Lateral Earth Pressure For At Rest Condition

The earth retained behind retaining walls may be natural earth or filled up soil. These backfill materials exert certain lateral pressures on the wall. If the wall is rigid and does not move with the pressure exerted on the wall, the soil behind the wall will be in a state of *elastic equilibrium* .

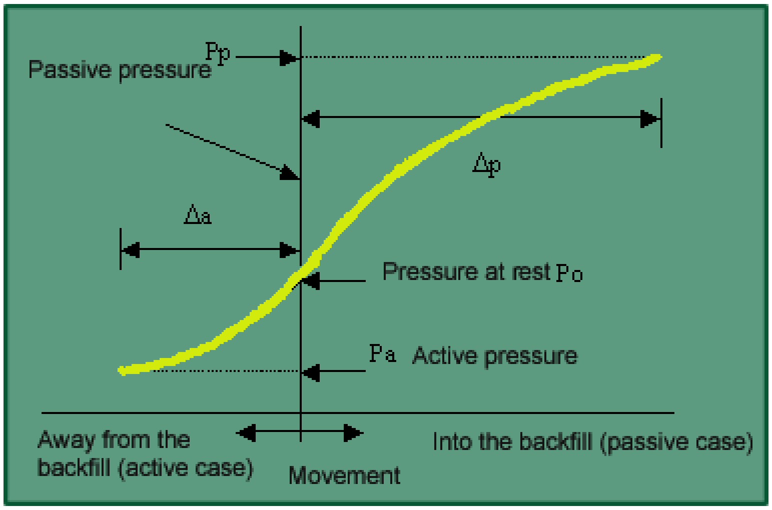


Fig.2.3 Development of active and passive earth pressures

To reach the plastic state, for p_a to develop we need a minimum displacement δ_a. If the displacement developed is less than δ_a, the pressure developed is called partially mobilized earth pressure which is more than p_a.

Similarly for passive case the wall moves towards the soil and the soil tries to resist the movement of the wall. To reach the passive case a very high movement is required, δ_p .

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.1 Different Theories of Earth Pressure]

When we are measuring the passive pressure we are measuring the resistance of the soil against wall movement.

Hence for all practical purposes we get fully mobilised active earth pressure but partially mobilised passive earth pressure as such a large movement does not occur usually. Hence it is more acceptable to design with active earth pressure.

Thus when δ_a is reached active pressure is fully mobilised but passive pressure is partially mobilised. Therefore we should design with (p_a – a small % of p_p), as, we are making an unsafe design when we are designing with
(p_a – fully mobilised p_p) and an uneconomic one when we are designing with only p_a.

To get fully mobilised active and passive earth pressures the following wall movement are required approximately.

δ_a = (0.1 - 0.4)% of the height of the wall.

δ_p = (5-10)% of the height of the wall.

For dense sand p_a mobilised at 0.1% strain and p_p at 5% strain.

For loose sand p_a mobilised at 0.4% strain and p_p at 10% strain.

Module 2 :Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.1 Different Theories of Earth Pressure]

2 Movement of the Wall

Considering a more generalised case when wall is not smooth.

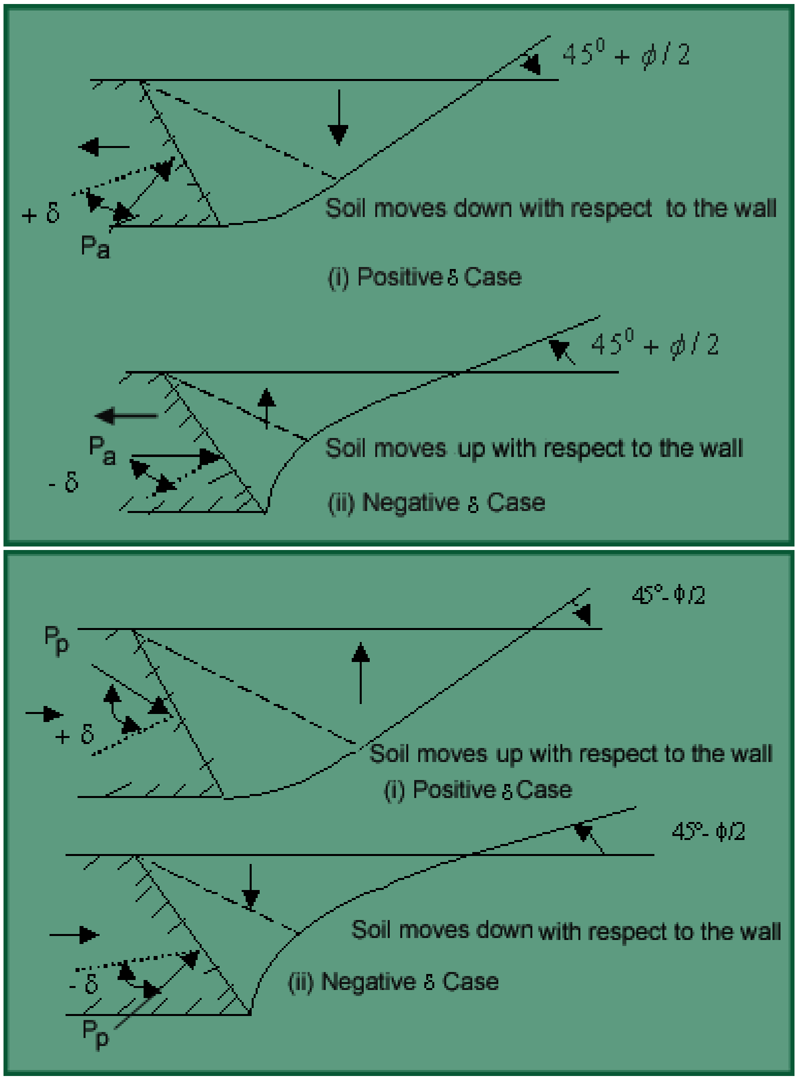


Fig. 2.4 Different movements of the wall

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.1 Different Theories of Earth Pressure]

3 Different Movements

There are basically three ways a wall can move. They are as follows:-

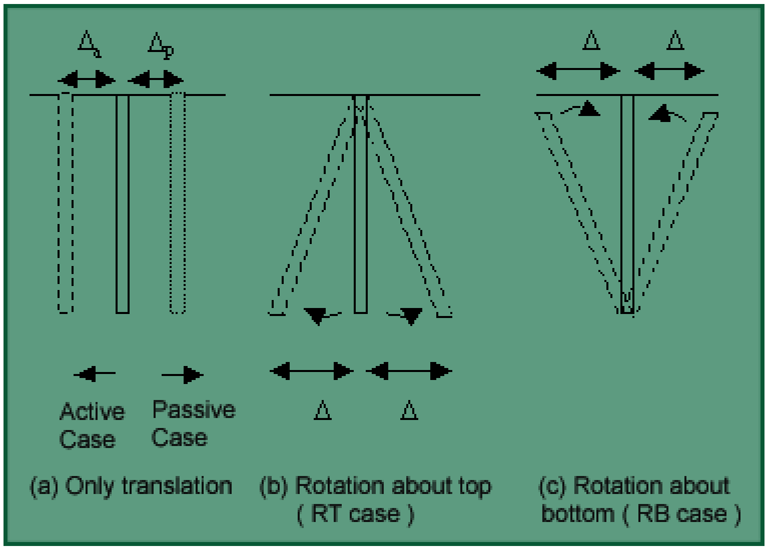


Fig. 2.5 Movements of the wall for both active and passive cases

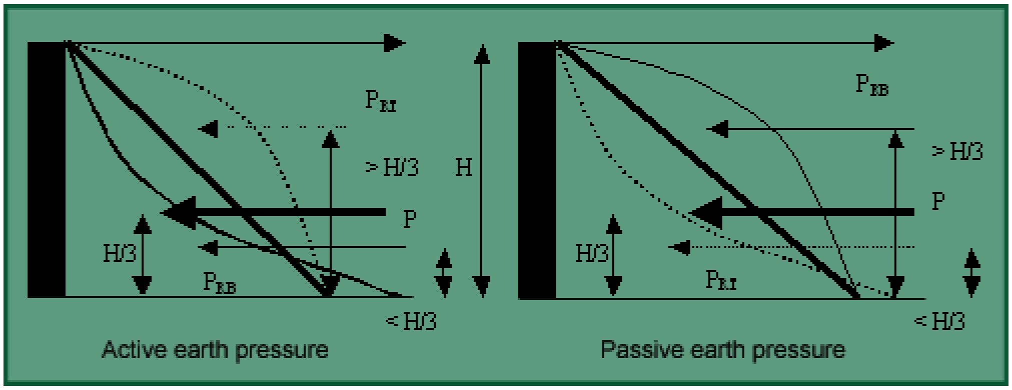
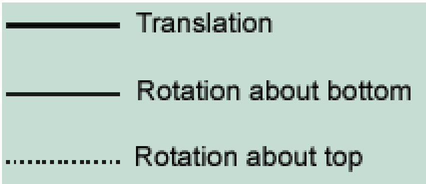


Fig. 2.6 Earth pressure distribution



Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.1 Different Theories of Earth Pressure]

Recap

In this section you have learnt the following

- Introduction
- Different Theories of Earth Pressure
- Lateral Earth Pressure For At Rest Condition
- Movement of the Wall
- Different Movements

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.2 Displacement Related Earth Pressure]

Objectives

In this section you will learn the following

- Displacement Related Earth Pressure
- Displacement-related active earth pressure
- Types of movements
- Proposed method
- Translation
- Rotation about top (RT)
- Rotation about the bottom (RB)

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.2 Displacement Related Earth Pressure]

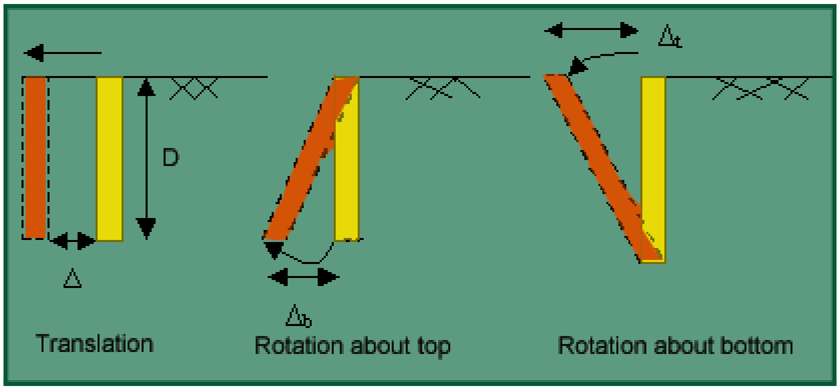
6.2 Displacement Related Earth Pressure

1. Displacement-related active earth pressure

Types of movements

Three types of rigid body movements are considered in the analysis. These are translation, rotation about top (RT) and rotation about bottom (RB) modes as shown in Fig. 2.7 below.

Case 1: Active earth pressure



Case 2: Passive earth pressure

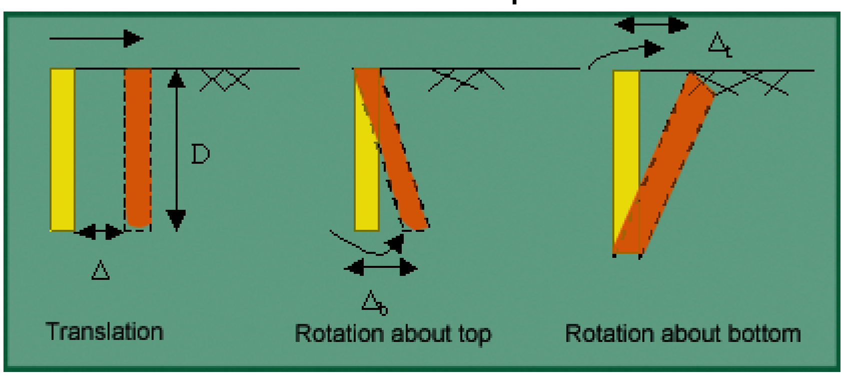


Fig 2.7 different movements of the wall under active and passive conditions

Δ is the displacement at any depth Z and Δ_a is the amount of displacement required for full mobilization of active earth pressure. Δ_a is found to be 0.1% to 0.4% of D for dense to loose sands where D is the total height of the retaining wall. The displacement at the base of the wall is denoted by Δ_b for RT mode and the displacement at the top of the wall is denoted by Δ_t for RB mode.

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction[Section 6.2 Displacement Related Earth Pressure]

2. Proposed method:

In the proposed semi-empirical approach, the assumptions made are:

- For translation mode, when $\Delta = \Delta_a$ full active pressure gets mobilized at all depths. So Δ is assumed to be constant throughout the depth.
- For rotation about top (RT) mode, Δ is a linear function of depth z. At the bottom when $\Delta_b = \Delta_a$, the mobilized friction angle ϕ_m will be zero at the top and full value of ϕ at the bottom. For $\Delta_t < \Delta_a$ ϕ_m < ϕ at the bottom and ϕ_m = 0at the top.
- For rotation about bottom mode, Δ is also a linear function of depth Z. At top when $\Delta_t = \Delta_a$ the mobilized friction angle ϕ_m will be zero at bottom and full value of ϕ at the top. For $\Delta_t < \Delta_a$ ϕ_m < ϕ at the top and ϕ_m = 0 at the bottom.
- The mobilized wall friction angle δ_m increases with the increase in mobilized friction angle resulting in a constant value of δ_m / ϕ_m

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction[Section 6.2 Displacement Related Earth Pressure]

I. Translation

The fig. 2.5 (a) shows a rigid retaining wall of height D. The amount of displacement is denoted by Δ, which is independent of depth z. The steps followed are,

- The experimental observations by Sherif et al (1984) and Fang et al (1986) are used. At various depths, for different wall movements, normal active earth pressure σ_{ax} is noted from experimental curves.
- The mobilized active earth pressure coefficient $(K_a)_m$ at any depth z is calculated as $(K_a)_m = \dfrac{\sigma_{ax}}{\gamma Z}$ Where σ_{ax} is the measured normal pressure at any depth z and γ is unit weight of soil.
- Along the depth, average $(K_a)_m$ is calculated and denoted by $(K_a)_{m.avg}$. Some abnormal values very near to ground are neglected because of the arching action and also some experimental error near the ground as reported by the experimentalists in their paper.
- Using Coulomb's active earth pressure coefficient (Bowles, 1996), mobilized friction angle ϕ_m is obtained corresponding to $(K_a)_{m.avg}$.
- The above procedure is repeated for various values of Δ / Δ_a. In table 2.1, a typical calculation is shown.

Table 2.1 : Typical calculation to obtain mobilized friction angle in translation mode

$\left(\phi = 34^0, \delta / \phi = 0.5, \gamma = 15.4 kN/m^3 and \Delta / \Delta_a = 1.0 with \Delta_a = 0.0005D\right)$ (Data from Fang et al, 1986),

Depth z in m	**Horizontal active earth pressure σ_{ax} in** kN/m ^{2}	$(K_a)_m$ values $\sigma_{ax} / \gamma z$	$(K_a)_{m.avg}$ (neglecting 1^{st} value)	Calculated ϕ_m (Degrees)	Corrected ϕ_m (Degrees)
0	0	-	-	-	-
0.153	1.166	0.497	-	-	-
0.305	1.114	0.237	-	-	-
0.467	1.531	0.213	-	-	-
0.628	2.353	0.243	0.204	38.29^0	34^0
0.793	2.281	0.187	-	-	-
0.958	2.541	0.172	-	-	-
1.016	2.645	0.169	-	-	-

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction[Section 6.2 Displacement Related Earth Pressure]

Now in case of full mobilization(Δ / Δ_a), $\phi_m = \phi$, hence a correction factor of (34/38.29) is applied. In this way all other data for partial mobilization case are also calculated. In other words, for any other (Δ/Δ_a), the corrected ϕ_m = (the calculated ϕ_m for any Δ) * (the correction factor).

Finally all results are expressed in terms of two parameters viz.

$x = \dfrac{\Delta}{\Delta_a}$ y = [(ϕ_m / ϕ) /(Δ / Δ_a)]

$y = \dfrac{1}{x^{0.65}}$ valid for 0<x<=1.

II Rotation about top (RT)

Consider fig. 2.5 (b) a rigid retaining wall of height D is rotating about the top and the bottom horizontal displacement is denoted by Δ_b Let,

$\xi = \Delta_b / \Delta_a$

assuming linear variation with depth,

$x = \Delta / \Delta_a = \xi(Z/D)$

$y = \left[\left(\phi_m/\phi\right)/\left(\Delta/\Delta_a\right)\right]$

The values of x and y thus obtained for various cases are shown in table 2.2 and it is found that all the points are falling within a narrow band with each point as an average of multiple points in one experiment.

A lower bound equation will provide a proper safe estimate of active earth pressure and hence the proposed relationship for translation mode is expressed as given below.

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction[Section 6.2 Displacement Related Earth Pressure]

The following Table 2.2 shows a typical calculation. In the full mobilization case, at the base $\Delta_b = \Delta_a$, so $\phi_m = \phi$ at the base and at the top $\phi = 0^0$. The correction factor at the base is therefore (40.4/71.47)=0.565, and at the top it is zero. By assuming a linear variation of correction factor from the top, the calculated values of ϕ_m are corrected as shown in table For other cases with $\Delta_b < \Delta_a$, the correction factor of 0.565 obtained at the base for the case of $\Delta_b = \Delta_a$ is maintained, with again a linear variation from zero at the top.

Some ill-points very near to the ground are neglected by considering that experimental errors may have been caused due to some arching action near ground as also mentioned by experimentalists in their papers.

Table 2.2 :Typical calculation to obtain mobilized friction angle in RT mode
($\phi = 40.4^0, \delta/\phi = 0.5, \gamma = 16.1 kN/m^3$ and $\Delta/\Delta_a = 1.0$ with $\Delta_a = 0.001D$) (Data from Fang et al, 1986),

Depth z in m	**Horizontal acticve earth pressure σ_{ax} in kN/m ^{2}**	$(K_a)_m$ values $\sigma_{ax}/\gamma z$	Calculated ϕ_m (Degrees)	Z/D values $(=\Delta/\Delta_b)$	Corrected ϕ_m (Degrees)	$x = \Delta/\Delta_a$ $[=1.0(\Delta/\Delta_b)]$	$y = [(\phi_m/\phi)/(\Delta/\Delta_a)]$
0	0	-	-	0	0	0	-
0.153	4.790	1.951	-	0.15	-	0.15	-
0.305	1.294	0.263	32.54	0.3	23.12	0.3	1.91
0.467	1.444	0.192	39.50	0.46	25.22	0.46	1.36
0.628	1.87	0.185	40.32	0.62	21.1	0.62	0.84
0.793	1.583	0.124	48.51	0.78	24.25	0.78	0.77
0.958	0.766	0.05	63.32	0.94	34.02	0.94	0.9
1.016	0.298	0.018	71.47	1.00	40.40	1.00	1.00

The proposed equation as a lower bound is given by,

$$y = \frac{1}{x^{0.5}} \qquad \text{-------(1)}$$

and eq(1) is valid for 0<x<=1 and 0< ξ <1. However at depth z=0, for any value of ξ, active earth pressure will be zero (ground surface).

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction[Section 6.2 Displacement Related Earth Pressure]

III Rotation about the bottom (RB)

Consider Fig.2.5 (c), a rigid retaining wall of height D, rotating about the bottom and the horizontal displacement at top is given by Δ_t, Let,

$$\eta = \Delta_t / \Delta_a \qquad \text{----------(2)}$$

and assuming linear variation with depth,

$$X = (1 - \Delta / \Delta_a) = 1 - \eta(1 - Z / D) \qquad \text{----------(3)}$$

$$Y = [(\phi_m / \phi)/(1 - \Delta / \Delta_a)] \qquad \text{----------(4)}$$

Table 2.3 : Typical calculation to obtain mobilized friction angle in RB mode

$(\phi = 35^o, \delta / \phi = 0.58, \gamma = 15.5 kN/m^3$ and $\Delta / \Delta_a = 1.0$ with $\Delta_a = 0.0003D)$ (Data from Fang et al, 1986),

Depth z in m	**Horizontal acticve earth pressure σ_{ax} in kN/m ^{2}**	$(K_a)_m$ values $\sigma_{ax} / \gamma z$	Calculated ϕ_m (Degrees)	1-Z/D values (Δ / Δ_t)	Corrected ϕ_m (Degrees)	$X = 1 - \Delta / \Delta_a$ $[= 1 - (\Delta / \Delta_a)]$	$Y = \left[\frac{(\phi_m / \phi)/}{(1 - \Delta / \Delta_a)} \right]$
0	0	-	36.3	1.00	35.00	0.00	-
0.153	0.559	0.237	34.39	0.83	27.55	0.17	4.72
0.302	1.331	0.284	30.10	0.67	17.82	0.33	1.54
0.467	1.663	0.23	34.98	0.49	16.72	0.51	0.94
0.628	2.461	0.253	32.93	0.31	8.79	0.69	0.37
0.793	2.994	0.244	33.76	0.13	3.63	0.87	0.12
0.915	3.326	0.235	34.57	0.00	0.00	1.00	0.00

Table 2.3 shows a typical calculation. In the full mobilization case at top $(\Delta_t / \Delta_a = 1)$, ϕ_m = ϕ at the top and at the base ϕ = 0 ^{0}. Considering this criteria, correction has been made using a correction factor of (35/36.3) at the top. Corrected values ϕ_m are shown in table 2.3. For other cases with $\Delta_t < \Delta_a$, the same maximum correction factor is applied at top to a linear variation of zero at the base.

The proposed equation as a lower bound to the points is given by,

$$Y = (1/x) - 1 \qquad \text{--------(5)}$$

And Eq.(5) is valid for 0<X<=1 and η not equal to zero.

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 6 : Introduction [Section 6.1 Different Theories of Earth Pressure]

Recap

In this section you have learnt the following

- Displacement Related Earth Pressure
- Displacement-related active earth pressure
- Types of movements
- Proposed method
- Translation
- Rotation about top (RT)
- Rotation about the bottom (RB)

Congratulations, you have finished Lecture 6. To view the next lecture select it from the left hand side menu of the page

Objectives

In this section you will learn the following

- Earth Pressure Theories
- Rankine's Earth Pressure Theory
- Active earth pressure
- Passive earth pressure
- Coulomb's Wedge Theory

7.1 Earth Pressure Theories

1 Rankine's Earth Pressure Theory

The Rankine's theory assumes that there is no wall friction $(\delta = 0)$, the ground and failure surfaces are straight planes, and that the resultant force acts parallel to the backfill slope.

In case of retaining structures, the earth retained may be filled up earth or natural soil. These backfill materials may exert certain lateral pressure on the wall. If the wall is rigid and does not move with the pressure exerted on the wall, the soil behind the wall will be in a state of *elastic equilibrium* . Consider the prismatic element E in the backfill at depth, z, as shown in Fig. 2.8.

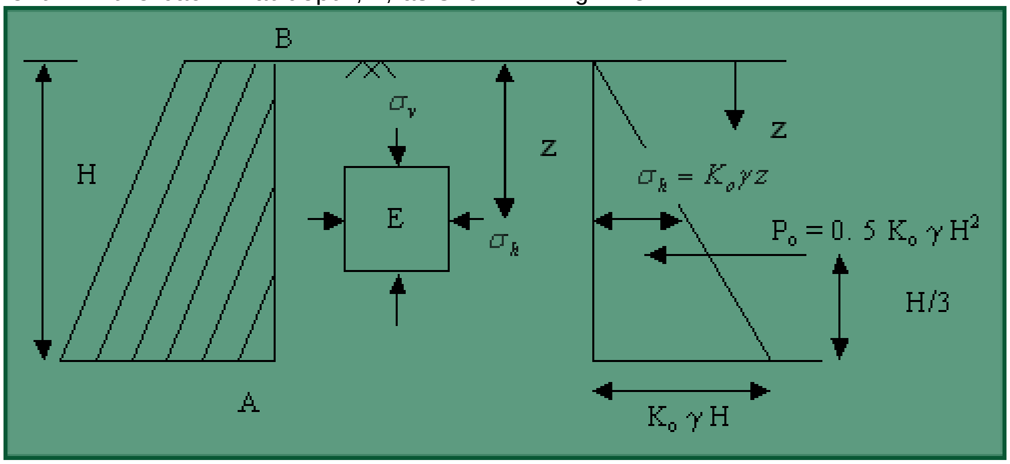


Fig. 2.8 Lateral earth pressure for at rest condition.

The element E is subjected to the following pressures :

Vertical pressure = $\sigma_v = \gamma z$

Lateral pressure = σ_h, where g is the effective unit weight of the soil.

If we consider the backfill is homogenous then both $\sigma_v and\ \sigma_h$ increases rapidly with depth z. In that case the ratio of vertical and lateral pressures remain constant with respect to depth, that is $\sigma_h / \sigma_v = \sigma_h / \gamma z$=constant =$K_o$, where $K_o$is the coefficient of earth pressure for at rest condition.

At rest earth pressure

The at-rest earth pressure coefficient (K_o) is applicable for determining the active pressure in clays for strutted systems. Because of the cohesive property of clay there will be no lateral pressure exerted in the at-rest condition up to some height at the time the excavation is made. However, with time, creep and swelling of the clay will occur and a lateral pressure will develop. This coefficient takes the characteristics of clay into account and will always give a positive lateral pressure.

The lateral earth pressure acting on the wall of height H may be expressed as $\sigma_h = K_o \gamma H$.

The total pressure for the soil at rest condition, $P_o = 0.5 K_o \gamma H^2$.

The value of K_o depends on the relative density of sand and the process by which the deposit was formed. If this process does not involve artificial tamping the value of K_o ranges from 0.4 for loose sand to 0.6 for dense sand. Tamping of the layers may increase it upto 0.8.

From elastic theory, $K_o = \mu/(1-\mu)$. ,where μ is the poisson's ratio.

According to Jaky (1944), a good approximation of K_o is given by , $K_o = 1 - \sin\phi$.

Table 2.4 : Different values of K_o

Soil Type	Typical Value for Poisson's Ratio	K_o
Clay, saturated	0.40 - 0.50	0.67 - 1.00
Clay, unsaturated	0.10 - 0.30	0.11 - 0.42
Sandy Clay	0.20 - 0.30	0.25 - 0.42
Silt	0.30 - 0.35	0.42 - 0.54
Sand - Dense	0.20 - 0.40	0.25 - 0.67
- Coarse (valid upto 0.4 - 0.7)	0.15	0.18
- Fine-grained (valid upto 0.4 - 0.7)	0.25	0.33
Rock	0.10 - 0.40	0.11 - 0.67

Rankine's Earth Pressure Against A Vertical Section With The Surface Horizontal With Cohesionlesss Backfill

Active earth pressure:

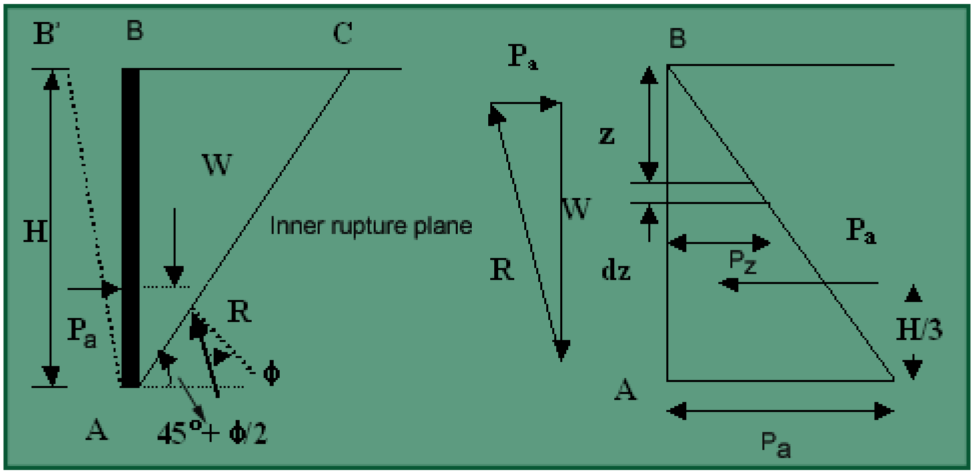


Fig. 2.9 Rankine's active earth pressure in cohesionless soil

The lateral pressure acting against a smooth wall AB is due to mass of soil ABC above the rupture line AC which makes an angle of $\left(45^0 + \phi/2\right)$ with the horizontal. The lateral pressure distribution on the wall AB of height H increases in same proportion to depth.

The pressure acts normal to the wall AB.

The lateral active earth pressure at A is $P_a = K_A \gamma H$, which acts at a height H/3 above the base of the wall. The total pressure on AB is therefore calculated as follows:

$$P_a = \int_0^H p_z dz = \int_0^H K_A \gamma z dz = 0.5 K_A \gamma H^2, \text{where } K_A = \tan^2\left(45^0 + \phi/2\right)$$

Passive earth pressure:

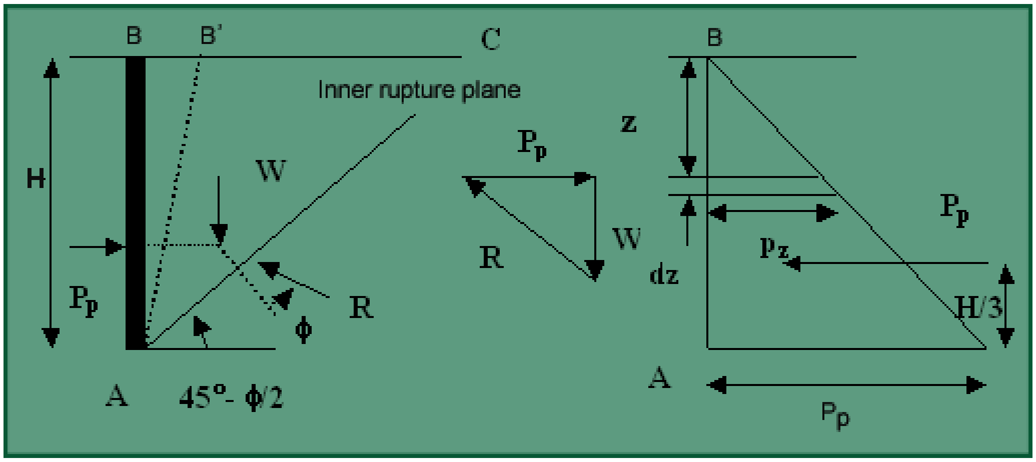


Fig. 2.10 Rankine's passive earth pressure in cohesionless soil

If the wall AB is pushed into the mass to such an extent as to impart uniform compression throughout the mass, the soil wedge ABC in fig. will be in Rankine's Passive State of plastic equilibrium. The inner rupture plane AC makes an angle $\left(45^0 + \phi/2\right)$ with the vertical AB. The pressure distribution on the wall is linear as shown.

The lateral passive earth pressure at A is $P_p = K_p \gamma H$. which acts at a height H/3 above the base of the wall. The total pressure on AB is therefore

$$P_p = \int_0^H p_z dz = \int_0^H K_p \gamma z dz = 0.5 K_p \gamma H^2, \text{ where } K_p = \tan^2\left(45^0 + \phi/2\right)$$

Rankine's active earth pressure with a sloping cohesionless backfill surface

Fig shows a smooth vertical gravity wall with a sloping backfill with cohesionless soil. As in the case of horizontal backfill, active case of plastic equilibrium can be developed in the backfill by rotating the wall about A away from the backfill. Let AC be the plane of rupture and the soil in the wedge ABC is in the state of plastic equilibrium.

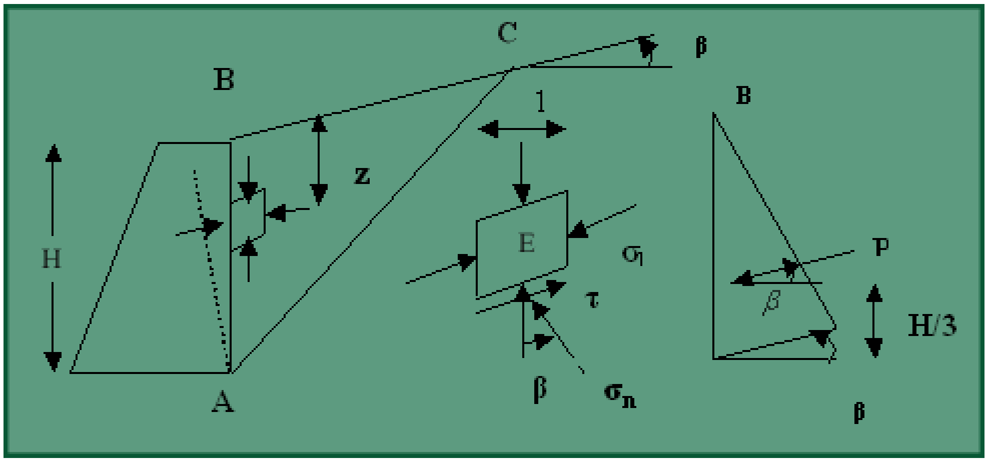


Fig. 2.11 Rankine's active pressure for a sloping cohesionless backfill

The pressure distribution on the wall is shown in fig. The active earth pressure at depth H is $P_a = K_A \gamma H$ which acts parallel to the surface. The total pressure per unit length of the wall is $P_a = 0.5 K_A \gamma H^2$ which acts at a height of H/3 from the base of the wall and parallel to the sloping surface of the backfill. In case of active pressure,

$$K_A = \cos\beta\left(\cos\beta - \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right) / \left(\cos\beta + \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right).$$

In case of passive pressure,

$$K_p = \cos\beta\left(\cos\beta + \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right) / \left(\cos\beta - \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right)$$

Rankine's active earth pressures of cohesive soils with horizontal backfill on smooth vertical walls

In case of cohesionless soils, the active earth pressure at any depth is given by

$P_a = K_A \gamma z$ In case of cohesive soils the cohesion component is included and the expression becomes $P_a = K_A \gamma z - 2c\sqrt{K_A}$.

When $P_a = 0, z = z_o = (2c\sqrt{K_A}) / \gamma$.

This depth is known as the depth of tensile crack. Assuming that the compressive force balances the tensile force (-), the total depth where tensile and compressive force neutralizes each other is $2z_o$. This is the depth upto which a soil can stand without any support and is sometimes referred as the depth of vertical crack or critical depth $(H_c)(H_c = 4c\sqrt{K_A}) / \gamma$.

However Terzaghi from field analysis obtained that $(H_c = 4c\sqrt{K_A}) / \gamma - z_o$, where,

$z_o \approx H_c / 2$ and is not more than that.

The Rankine formula for passive pressure can only be used correctly when the embankment slope angle equals zero or is negative. If a large wall friction value can develop, the Rankine Theory is not correct and will give less conservative results. Rankine's theory is not intended to be used for determining earth pressures directly against a wall (friction angle does not appear in equations above). The theory is intended to be used for determining earth pressures on a vertical plane within a mass of soil.

2 Coulomb's Wedge Theory

The Coulomb theory provides a method of analysis that gives the resultant horizontal force on a retaining system for any slope of wall, wall friction, and slope of backfill provided. This theory is based on the assumption that soil shear resistance develops along the wall and failure plane. The following coefficient is for resultant pressure acting at angle. Since wall friction requires a curved surface of sliding to satisfy equilibrium, the Coulomb formula will give only approximate results as it assumes planar failure surfaces. The accuracy for Coulomb wil1 diminishe with increased depth. For passive pressures the Coulomb formula can also give inaccurate results when there is a large back slope or wall friction angle. These conditions should be investigated and an increased factor of safety considered.

Coulomb (1776) developed a method for the determination of the earth pressure in which he considered the equilibrium of the sliding wedge which is formed when the movement of the retaining wall takes place. The sliding wedge is torn off from the rest of the backfill due to the movement of the wall. In the Active Earth Pressure case, the sliding wedge moves downwards & outwards on a slip surface relative to the intact backfill & in the case of Passive Earth pressure, the sliding wedge moves upward and inwards. The pressure on the wall is, in fact, a force of reaction which it has to exert to keep the sliding wedge in equilibrium. The lateral pressure on the wall is equal and opposite to the reactive force exerted by the wall in order to keep the sliding wedge in equilibrium. The analysis is a type of limiting equilibrium method.

The following assumptions are made

- The backfill is dry, cohesion less, homogeneous, isotropic and ideally plastic material, elastically undeformable but breakable.
- The slip surface is a plane surface which passes through the heel of the wall.
- The wall surface is rough. The resultant earth pressure on the wall is inclined at an angle δ to the normal to the wall, where δ is the angle of the friction between the wall and backfill.
- The sliding wedge itself acts as a rigid body & the value of the earth pressure is obtained by considering the limiting equilibrium of the sliding wedge as a whole.
- The position and direction of the resultant earth pressure are known. The resultant pressure acts on the back of the wall at one third height of the wall from the base and is inclined at an angle δ to the normal to the back. This angle is called the angle of wall friction.
- The back of the wall is rough & relative movement of the wall and the soil on the back takes place which develops frictional forces that influence the direction of the resultant pressure.

In coulomb's theory, a plane failure is assumed and the lateral force required to maintain the equilibrium of the wedge is found using the principles of statics. The procedure is repeated for several trial surfaces. The trial surface which gives the largest force for the active case, and the smallest force for the passive case, is the actual failure surface. This method readily accommodates the friction between the wall & the backfill, irregular backfill, sloping wall, & the surcharge loads etc. Although the initial theory was for dry, cohesion less soil it has now been extended to wet soils and cohesive soils as well. Thus Coulomb's theory is more general than the Rankine's Theory.

Coulomb's Active Pressure in cohesionless soils

Fig 2.12 shows a retaining wall with an inclined back face and sloping dry granular backfill. In active case, the sliding wedge ABD moves downward, and the reaction R acts upward and inclined at an angle f' with the normal.

The sliding wedge ABD is in equilibrium under the three forces:

- Weight of the wedge (W).
- Reaction R on the slip surface BD.
- Reaction P_a from the wall (wall reaction)/Earth pressure.

For the condition of the yield of the wall away from the backfill, the most dangerous or the critical slip surface is that for which the wall reaction is maximum, i.e., the wall must resist the maximum lateral pressure before it moves away from the backfill; the lateral pressure under this condition is the active pressure. The critical slip surface for the case of the passive earth pressure is that for which the wall has to exert a minimum force to tear off a soil wedge by moving towards the backfill.

The main deficiency of this theory is the assumptions that the slip surface is planar, therefore, the force acting on the slide wedge do not generally meet when in static equilibrium condition. The actual slip surface is curved, especially in the lower part. The assumptions of plane slip surface does not affect materially the results in the active case but give very high values in the passive case as compared with the assumptions of curved slip surfaces. The principle of the sliding wedge has been extended for calculating earth pressure of cohesive soils.

Fig 2.12 shows the force triangle. As the magnitude of one force (weight W) and the directions of all the three forces are known, the force triangle can be completed. The magnitude of P_a is determined from the force triangle. The pressure acting on the wall is equal and opposite of P_a

The procedure is repeated after assuming another failure surface. The surface that gives the maximum value of P_a is the critical failure plane; the corresponding force is the active force.

Coulomb's method does not give the point of application of the earth pressure (P_a).The point of application is found to be approximately at the point of intersection E of the back of the retaining wall with a line CE drawn from the centroid C of the failure wedge and parallel to the surface. As this procedure is cumbersome, for convenience, the pressure distribution is some times assumed to be hydrostatic on the back of the wall, and the resultant pressure P_a is assumed to act at one third the height of the wall from the base.

The following points should be carefully noted while using coulomb's theory:

- For most practical cases, the backfill moves down relative to the wall in the Active case, and, therefore, the active force P_a is inclined δ at an angle below the normal. However, if the wall is supported on a soft, compressible soil, it may settle to such extent that the movement of the wall be downward relative to the backfill and the relative movement of the wedge will be upward. In such a case, the force P_a would be inclined at an angle δ above the normal to the wall.
- The angle δ is the friction angle between the soil and the wall. It may be determined by means of a direct test. For concrete walls δ is generally taken as $(2/3)\phi'$. The value of δ can not exceed ϕ', because in that case the failure will occur in soil. If the friction angle δ is zero, for a vertical wall and the horizontal ground surface, the coulomb method gives identical results with Rankine Method.
- Coulombs method assumes the failure surface to be a plane. The actual failure surface is slightly curved. Fortunately, for the active case, the error is small, and the failure surface may be assumed to be planar without any significant error.

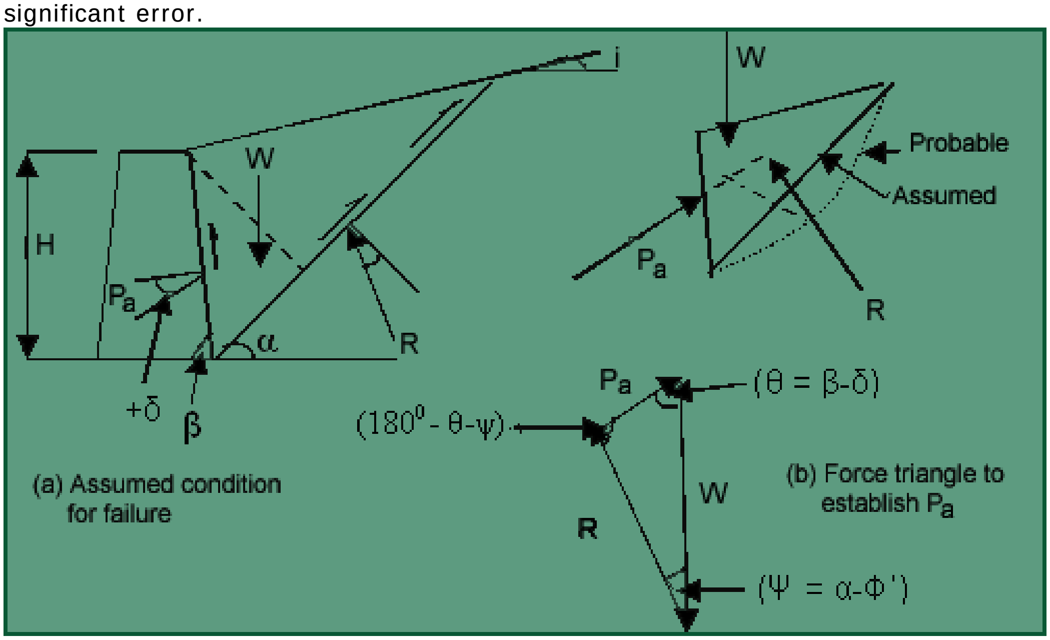


Fig. 2.12 Expression for Active Pressure for cohesion less soil

Using the law of sine
The active force is the component of the weight vector as from fig , we obtain

$$\frac{P_a}{\sin(\alpha - \phi')} = \frac{W}{\sin(180^0 - \beta - \alpha + \phi' - \delta)}$$

$$P_a = \frac{W.\sin(\alpha - \phi')}{\sin(180^0 - \beta - \alpha + \phi' + \delta)} \qquad \text{------------(a)}$$

Since from figure

$$BE = \frac{AB.\sin(\beta + i)}{\sin(\alpha - i)}$$

$$AG = AB.\sin(\alpha + \beta)$$

$$AB = \frac{H}{\sin\beta}$$

Therefore
The weight of the soil wedge is

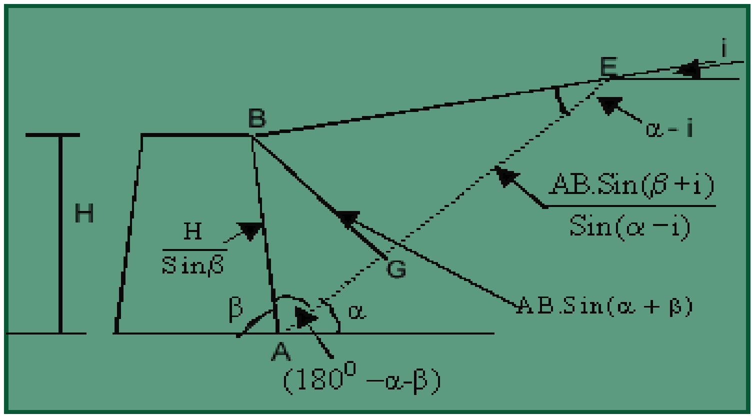


Fig. 2.13 Failure wedge

$$W = \frac{1}{2} \times BE \times AG \times \gamma \times 1$$

$$W = \frac{\gamma H^2}{2.\sin^2\beta}\left[\sin(\beta + \alpha)\right] \times \frac{\sin(\beta + i)}{\sin(\alpha - i)} \qquad \text{-----------(b)}$$

From eq. (a) it can be seen that the value of P_a = f (α); that is, all other terms for a given problem are constant, and the value of P_a of primary interest is the largest possible value. Combining the eq. (a) and (b), we obtain

$$P_a = \frac{\gamma H^2}{2.\sin^2\beta}\left[\sin(\beta+\alpha)\frac{\sin(\beta+i)}{\sin(\alpha-i)}\right].\frac{\sin(\alpha-\phi')}{\sin(180-\beta-\alpha+\phi'+\delta)} \qquad \text{---------(c)}$$

Equating the first derivative to zero, $\frac{dP_a}{d\alpha} = 0$

The maximum value of active wall Force P_a is found to be

$$P_a = \frac{\gamma H^2}{2}\times\frac{\sin^2(\beta+\phi')}{\sin^2\beta.\mathrm{Sin}(\beta-\delta)\left[1+\sqrt{\frac{\sin(\phi'+\delta).\sin(\phi'-i)}{\sin(\beta-\delta).\sin(\beta+i)}}\right]^2} \qquad \text{--------(d)}$$

If i = δ = 0 and ß = 90^0 (a smooth vertical wall with horizontal backfill)

Then Eq. (d) simplifies to

$$P_a = \frac{\gamma H^2}{2}\times\frac{1-\sin\phi'}{1+\sin\phi'} = \frac{\gamma H^2}{2}\tan(45-\frac{\phi'}{2}) \qquad \text{--------(e)}$$

This is also the Rankine equation for active earth pressure equation. Equation takes the general form

$$P_a = \frac{\gamma H^2}{2}.k_a$$

Where

$$K_a = \frac{\sin^2(\beta+\phi')}{\sin^2\beta.\sin(\beta-\delta)\left[1+\sqrt{\frac{\sin(\phi'+\delta).\sin(\phi'-\alpha)}{\sin(\beta-\delta).\sin(\beta+i)}}\right]^2}$$

K_a = coefficient which considered $\beta, i, \delta \; and \; \phi'$, but independent of γ and H.

Passive earth pressure is derived similarly except that the inclination at the wall and the force triangle will be as shown in Fig. 2.14.

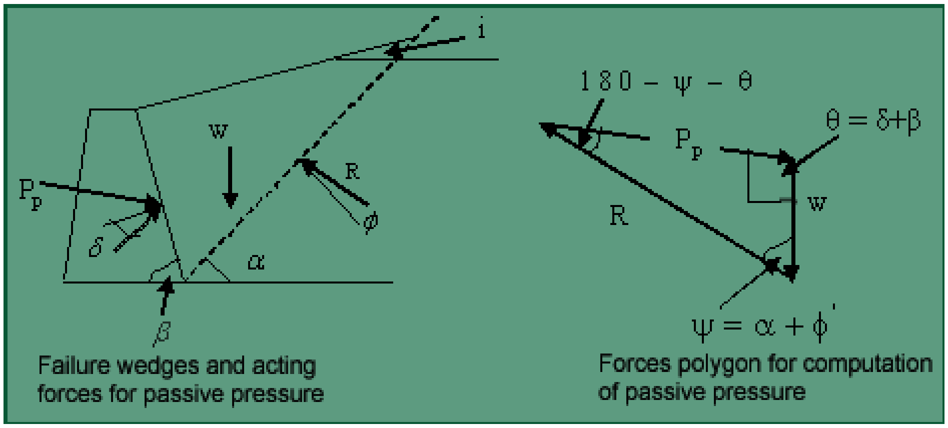


Fig. 2.14 Failure wedges

From above Fig. 2.14 the weight of the assumed failure mass is

$$W = \frac{\gamma . H^2}{2} \times \frac{\sin(\beta + i)}{\sin(\alpha - i)}$$

and from the force triangle, using the law of sines

$$P_p = W . \frac{\sin(\alpha + \phi')}{\sin(180 - \alpha - \phi' - \delta - \beta)}$$

Setting the derivative($\frac{dP_a}{d\alpha} = 0$) gives the minimum value of the P_p as

$$P_p = \frac{\gamma . H^2}{2} \times \frac{\sin^2(\beta - \phi')}{\sin^2\beta . \ in(\beta + \delta)\left[1 - \sqrt{\frac{\sin(\phi' + \delta).\sin(\phi' + i)}{\sin(\beta + \delta).\sin(\beta + i)}}\right]^2}$$

For a smooth vertical wall with the horizontal backfill (ß =90 ^{0}, δ = i = 0),

$$P_p = \frac{\gamma . H^2}{2} \times \frac{1 + \sin\phi'}{1 - \sin\phi'} = \frac{\gamma . H^2}{2} \times \tan(45 + \frac{\phi'}{2})$$

Equation can also be written as

$$P_p = \frac{\gamma H^2}{2} \times K_p$$

Where

$$K_p = \frac{\sin^2(\beta - \phi')}{\sin^2\beta . \sin(\beta + \delta)\left[1 - \sqrt{\frac{\sin(\phi' + \delta).\sin(\phi' + i)}{\sin(\beta + \delta).\sin(\beta + i)}}\right]^2}$$

Graphical solutions for lateral Earth Pressure

- Culman's solution
- The trial wedge method
- The logarithmic spiral

The Culmann solution

The Culmann's solution considered wall friction δ, irregularity of the backfill, any surcharges (either concentrated or distributed loads), and the angle of internal friction of the soil. Here we are describing the solution which is applicable to the cohesion less soils, although with some modification it can be used for the soils with cohesion. This method can be adapted for stratified deposits of varying densities, but the angle of internal friction must be the same throughout the soil mass. A rigid, plane rupture is assumed. Essentially the solution is a graphical determination of the maximum value of the soil pressure, and a given problem may have several graphical maximum points, of which the largest value is chosen as the design value. A solution can be made for both Active and Passive pressure.

Steps in Culmann's solution for Active Pressure are as follows:

- Draw the retaining wall to any convenient scale, together with the ground line, location of surface irregularities, point loads, surcharges, and the base of the wall when the retaining wall is a cantilever type.

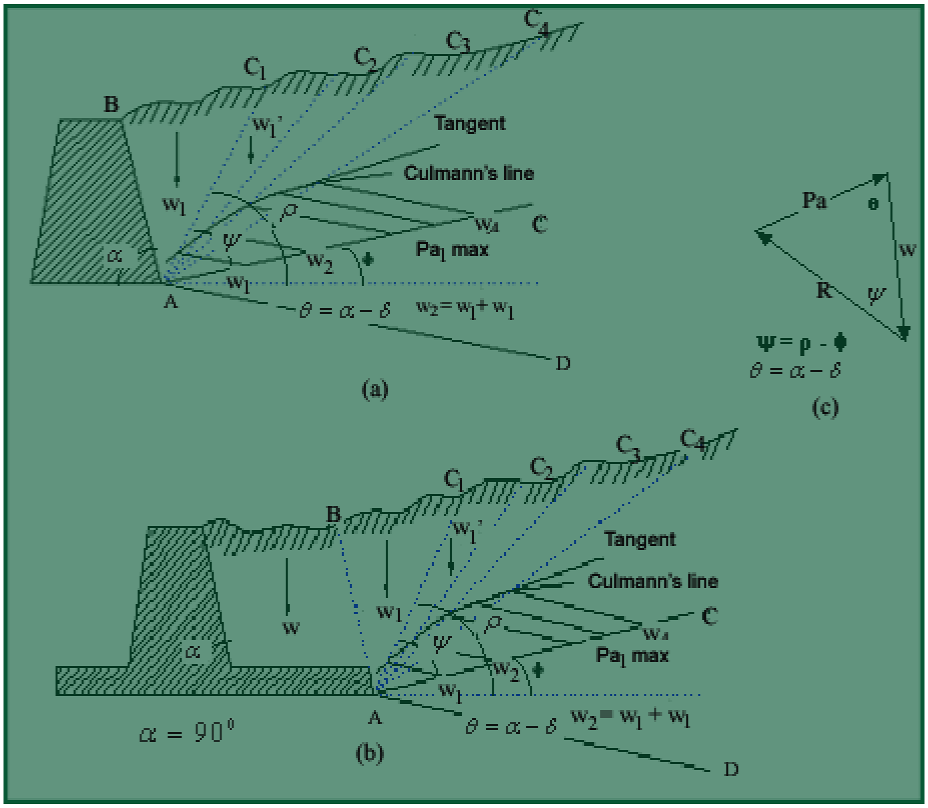


Fig 2.15a Graphical Solution By Culmann's Method

- From the point A lay off the angle ϕ with the horizontal plane, locating the line AC. Note that in the case of a cantilever wall, the point A is at the base of the heel, as shown in the fig. 2.15.
- Layoff the line AD at an angle of θ with line AC. The angle is $\theta = \alpha - \delta$.
  Where α = angle the back of the wall makes with horizontal, δ = angle of wall friction.
- Draw assumed failure wedges as $ABC_1, ABC_2, \ldots, ABC_n$. These should be made utilizing the backfill surface as a guide, so that geometrical shapes such as triangles and rectangles formed.

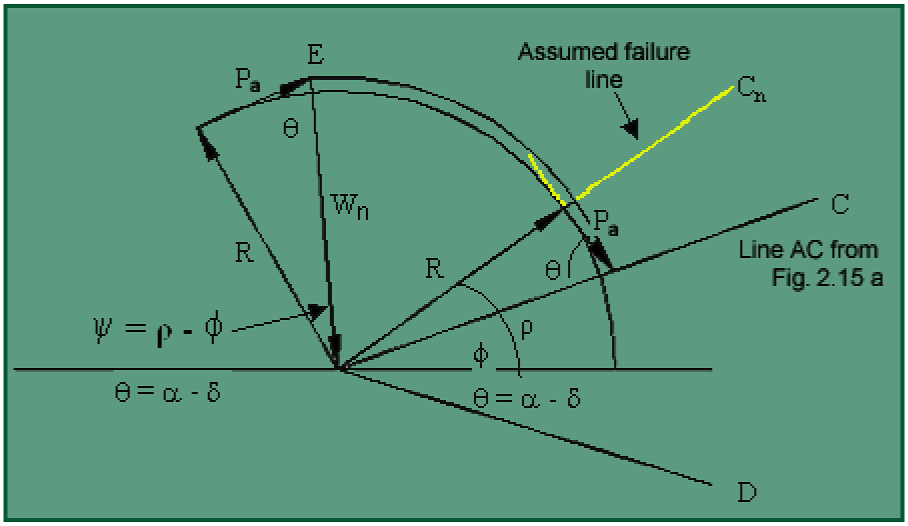


Fig. 2.15b. Analysis of Forces in Culmann's Method

- Find the weight W_n of each of the wedges by treating as triangles, trapezoidal or rectangles, depending on the soil stratification, water in the soil, and other condition and geometry.
- Along the line AC, plot to a convenient weight scale, the wedge weights, locating the points $w_1, w_2, \ldots\ldots, w_n$.
- Through the points just established in previous step, draw lines parallel to AD to intersect the corresponding side of the triangle, as W_1 to side AC_1, w_2 to side AC_1, AC_2, ..., w_n to side AC_n Through the locus of points established on the assumed failure wedges, draw a smooth curve (the Culmann line). Draw the tangent to this curve and parallel to the line AC .
- Through the tangent point established in above step, project a line back to the AC line, which is also parallel to the line AD. The value of this to the weight scale is P_a, and a line through the tangent point from A is the failure surface. When several tangents are drawn, choose the largest value of P_a .

The basis of the Culmann procedure is the solution of the force triangle shown in the figure. The triangle is rotated so that the location of the trial failure wedge automatically yields the angle ψ without recourse to measuring ψ each time. The line AD is laid off for use in projecting the instant value of P_a at the proper slope since θ is constant for a particular problem. The slope of R is automatically established from the slope of the weight line AC; thus, with all slopes and one side w_n known, the force triangle is readily solved.

To find the point of application of P_a, the following procedure (Terzaghi (1943)) is recommended.

Case1. No concentrated loads (fig a), but may have other surcharges.

- Find the centre of gravity of the failure wedge graphically or by trimming a cardboard model and hanging by a thread at the two or three points.
- Through the centre of gravity and parallel to the failure surface draw a line of action of P_a until it intercepts AB (wall or plane through the heel of the cantilever wall). P_a acts at an angle of δ or β to a perpendicular to AB.

Case 2. Concentrated load or line load within the failure wedge.(Fig b)

- Parallel to AC draw line Vc', and parallel to AC_f draw V c_f'.
- Take one- third of distance $c'c_f'$ from c' for the point of application of P_a.

Case 3. Concentrated load or line load outside the failure wedge (fig c).

- Draw a line from the concentrated load to A(V A).
- Draw V_c' parallel to Ac.
- Take one third of c'A from c' as point of application of P_a. If the surcharge falls out of Zone ABC, the problem should be treated as if no surcharge were present.

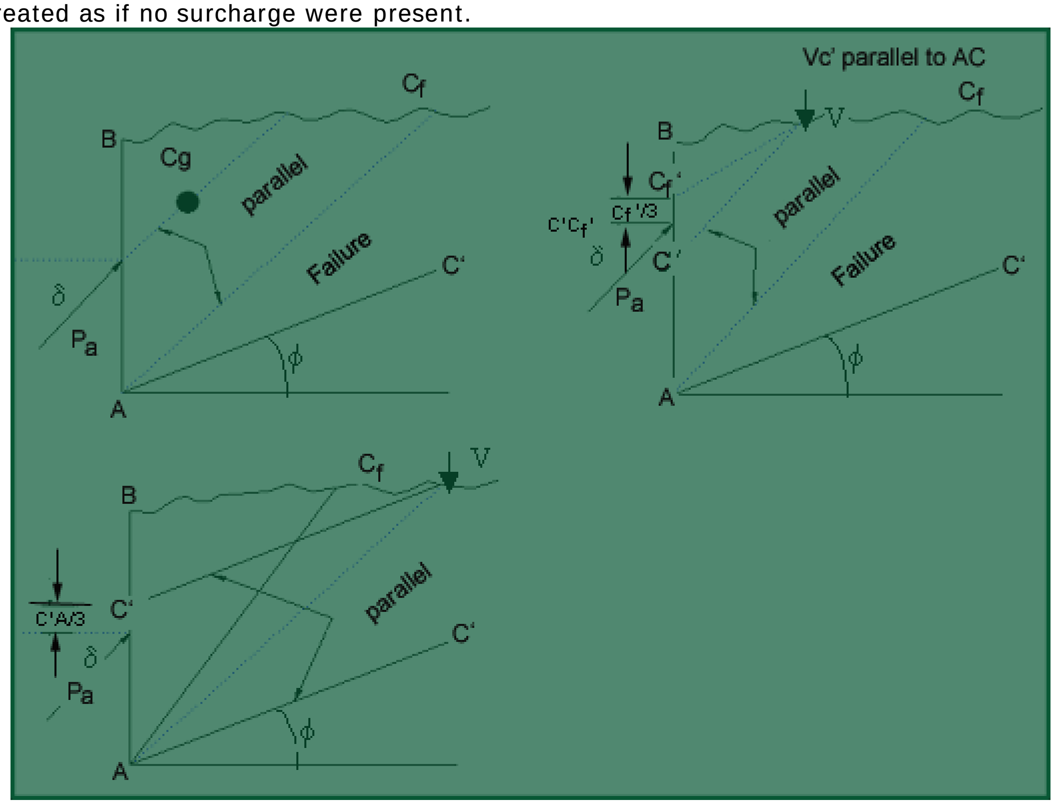


Fig. 2.16 Failure wedge

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 7 : Earth Pressure Theories [Section 7.1 Rankine and Coulomb Theory]

Recap

In this section you have learnt the following

- Earth Pressure Theories
- Rankine's Earth Pressure Theory
- Active earth pressure
- Passive earth pressure
- Coulomb's Wedge Theory

Objectives

In this section you will learn the following

- Friction Circle Method
- Terzaghi's Analysis
- Log-Spiral Theory

7.2.1 FRICTION CIRCLE METHOD

- This method is based on total stress analysis, but it enables the angle of shearing resistance to be taken into account.
- It should be noted that some soils, such as saturated silts and unsaturated clays, do exhibit a F value under un-drained conditions.

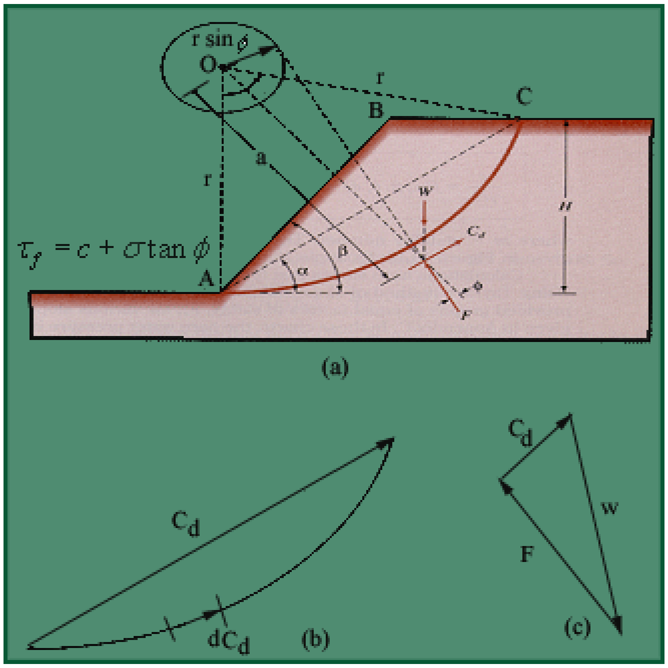


Fig. 2.17 Depiction of friction circle

- The friction circle method assumes a circular slip surface; it is in the form of a circular arc of radius R with its centre at 'O'.
- Let the slip surface arc AC (=L) be considered to be made up of a number of elementary arcs of length ΔL. The elementary cohesive force acting along this element of length ΔL opposing the probable movement of the soil mass is $dC_d L_d$. Where dC_d is the unit mobilized cohesion which is assumed to be constant along the slip surface. The total cohesive force $C_d = dC_d$ L is considered to be made up of elementary cohesive force $dC_d \Delta L$ representing a force polygon. The closing link of which must represent the magnitude and direction of the resultant cohesive force, which is equal to $dC_d L_d$.

- The position of the resultant $dC_d L_d$ is determined by equating the sum of the moments of all the elementary cohesive forces along the slip circle about the centre of rotation 'O' to the moment of the resistant about 'O'.
- $\tau_f = c' + \sigma' \tan\phi'$
  - $\overline{AC}$ is a trial circular arc that passes through the toe of the slope, and O is the center of the circle.
  - Weight of soil wedge ABC = W = (Area of ABC)(γ)
  - For equilibrium (Figure 2.17)

    C_d - resultant of the cohesive force

$$C_d = c'_d(\overline{AC})$$

$$C_d(a) = c'_d(AC)r$$

$$a = \frac{c'_d(AC)r}{C_d} = \frac{AC}{\overline{AC}}r$$

where 'a' is the moment arm of the cohesive force $dC_d AC$.

- If it is assumed that the frictional resistance is fully mobilized along the slip surface, the soil reaction dC_d on any elementary arc has its direction opposing the probable movement of the sliding mass and is inclined at ϕ to the normal at the point of action of dC_d on the slip surface. Line of action of dC_d will thus be a tangent to a circle of radius 'r sin ϕ',drawn with 'O' as centre. This circle is referred to as the *friction circle or the ϕ-circle.*
- The three forces considered in the equilibrium of the sliding mass can now be drawn to form a force triangle to determine the required value of dC_d from this force triangle, with the known values of 'W' and 'F' magnitudally with some direction , equal to $dC_d = C / L_d$.
- The factor of safety with respect to the cohesion is given by, $F_c = C_u / dC_d$. the minimum factor of safety is obtained by locating the critical slip circle. If the real factor of safety with respect to shear strength F s is required, a trial factor of safety with respect to friction F_ϕ is assumed. The friction circle is now constructed with a reduced radius 'r sin ϕ', where $\tan\phi_m = \tan\phi_u / F_\phi$. By carrying out the frictional circle analysis, the factor of safety with respect to cohesion, F_c is determined. If $F_c = F_\phi$, then this is the true factor of safety F_s . If not, a different F_ϕ is assumed and the procedure repeated till we obtain $F_\phi = F_c = F_s$.
- For an effective stress analysis, the total weight W is combined with the resultant boundary water force on the failure mass and the effective stress parameters c ' and ϕ' are used. This analysis is done to investigate the long form stability of a slope.

7.2.2 TERZAGHI'S ANALYSIS

The earth pressure increases rapidly with increasing values of internal friction ϕ . In the case of wall friction angle $\delta < \phi/3$, the error is around 5%. However, if δ is greater than about $\phi/3$, the surface of sliding is strongly curved. As a consequence, the error due to Coulomb's assumption of a plane surface increases rapidly. For $\delta = \phi$ it may be as great as 30 percent. Hence, for values of δ greater than $\phi/3$, the curvature of the surface of sliding must be taken into consideration. For passive pressures, the Coulomb formula can also give inaccurate results when there is a large back slope or wall friction angle. These conditions should be investigated and an increased factor of safety considered.

Log-Spiral Theory

A Log-spiral theory was developed because of the unrealistic values of earth pressures that are obtained by theories which assume a straight line failure plane. The difference between the Log-Spiral curved failure surface and the straight line failure plane can be large and on the unsafe side for Coulomb passive pressures (especially when wall friction exceeds $\phi/3$). The following figure shows a comparison of the Coulomb and Log-Spiral failure surfaces:

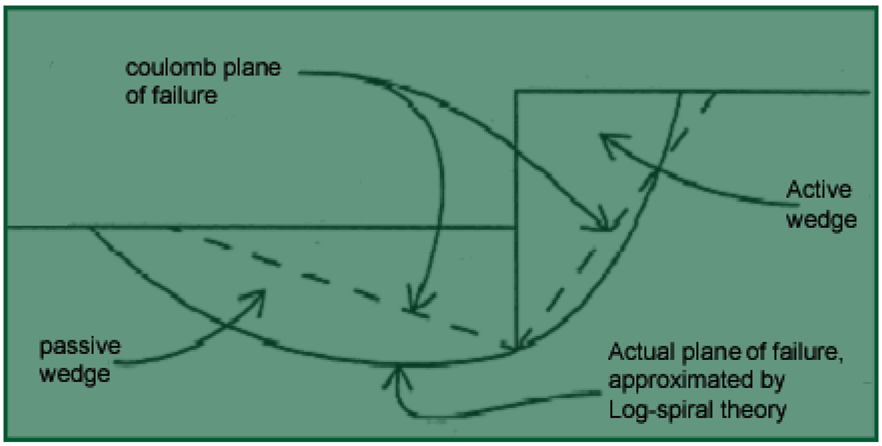


Fig. 2.18 Comparision of Coulomb's Theory and Log-Spiral Theory

Considering these factors, the theory put forward by Terzaghi is preferred for analyzing passive earth pressure when $\delta > \phi/3$. Terzaghi in 1943 published a theory for the computation of passive earth pressure.

Following are the assumptions made in this theory:

- Backfill is considered to be horizontal with a uniform surcharge density *q*. The shearing resistance of soil is given by the equation
  $s = c + \sigma \tan\phi$
- Rupture surface is assumed to be made up of a curved part *bd* and a straight line part *de* (fig 2.9). The soil within the triangle *ade* is in the passive Rankine state. Therefore, the shear stresses on the vertical section *df* are zero and the pressure on this section will be horizontal. The curve *bd* is taken as the log-spiral with the equation:
  $r = r_0 e^{\theta \tan\phi}$ ------------ (1)
  Centre of the log spiral is assumed to lie on the line *ad* at the tip *a*
- Three cases of loading are considered and the superposition of limit stresses for these three cases are considered to calculate the minimum resulting passive pressure.
  (a) *c*=*q*=0, $\gamma \neq 0$ (b) $c \neq 0$, $\gamma = q = 0$ (c) $q \neq 0$, $\gamma = c = 0$ holds good.
  where, *c*=cohesion, $\gamma =$ density of soil, *q*=intensity of surcharge.
  The following figure (2.19) illustrates the assumptions on which the theory of passive earth pressure against rough contact faces is based. Here, the lines *ab* and *ad* represents the initial (r_0) and final radius (r) of the log spiral respectively. The failure zone *abd* is thus divided into two parts viz ;(1) log spiral zone (2) Planar zone.

According to equation (1), every radius of the spiral makes an angle ϕ with the normal to the spiral at the point where it intersects the curve. Since ϕ is the angle of internal friction, the resultant *F* of the normal stress and the frictional resistance on any element of the surface of sliding also makes an angle ϕ with the normal to the element, and its direction coincides with that of the radius that the element subtends. Since every radius of the spiral passes through point a, the resultant F of the normal and frictional forces on *bd* also passes through the centre a.

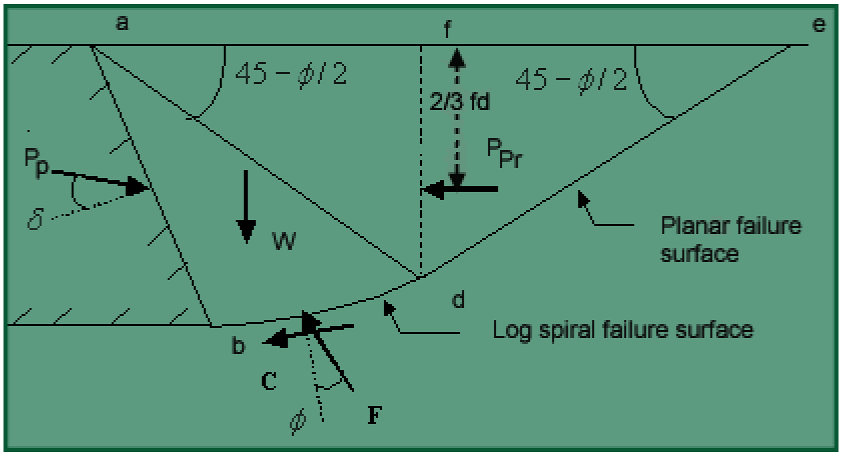


Fig. 2.19 Rupture Surface assumed in Terzaghi's Wedge Theory

Case (a): $c = q = 0, \gamma \neq 0$

In order to compute the passive pressure, we arbitrarily select a surface of sliding, *bde*

(**Figure** 2.21) consisting of the logarithmic spiral *bd* with its centre at *a*, and the straight line *de'*, which makes an angle of 45- $\phi/2$ with the horizontal. The surcharge q and the cohesion c are not taken into consideration in this case. The lateral pressure required to produce a slip on this surface is designated as P_p.

The Rankine's passive earth pressure P_{pr} which acts at the lower third-point of *fd*, is calculated as

$$P_{pr} = \frac{1}{2}\gamma H^2 N_\phi \text{ - } \qquad \text{---------(2)}$$

where H is the height of the wall, $N_\phi = \tan^2(45 + \phi/2)$

Taking moments of the forces about a,

$$P_p l_1 \cos\delta = W.l_2 + P_{PR}.l_3 \qquad \text{---------(3)}$$

Since the force F acts along the radius of the log spiral failure surface, its moment with respect to point a is zero.

The value of P_p is plotted to scale as $P_{p\gamma}$. This value corresponds to the failure surface *be'*. This process is repeated and the passive pressure for each assumed failure surface is found out. The curve resulting from the passive pressures corresponding to each failure surface is used to get the minimum passive pressure ($P_{p\gamma}$).

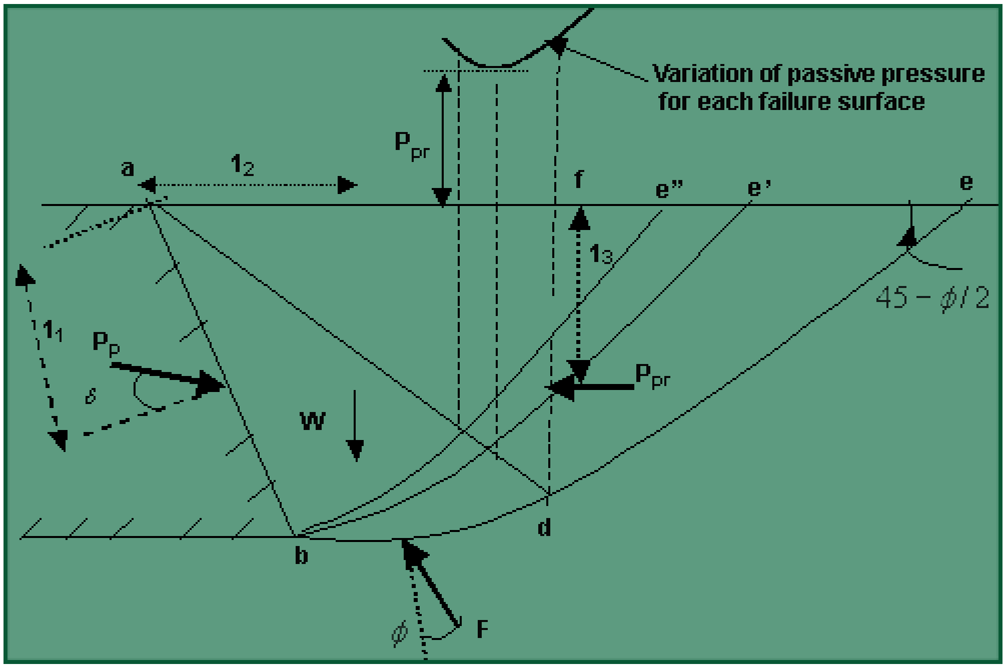


Fig. 2.20 Logarithmic Spiral Method for Determining Passive Earth Pressure

Case (b): $c \neq 0$, $\gamma = q = 0$

Here,the cohesive forces of the soil are used for the analysis while, the unit weight and surcharge of the soil are neglected. The adhesive force between the soil and the wall C_a is also taken into account.

The Rankine's passive pressure due to cohesion P_{PRC} is calculated as:

$$P_{PRC} = 2cH\sqrt{N_\varphi} \qquad \text{---------(4)}$$

Moment due to cohesion C, $M_c = \dfrac{c}{2\tan\phi}(r_f^{\,2} - r_0^{\,2})$ ---------(5)

The value of $\dfrac{c_a}{c}$ ranges from 0 to $\dfrac{\tan\delta}{\tan\phi}$. The shear strength corresponding to soil and wall are given as:

$\tau = c + \sigma\tan\phi$ (For soil) ----------(6)

$\tau = c_a + \sigma\tan\delta$ (For wall) ----------(7)

Adhesive force between the wall and the soil is given by

$$C_a = c_a.ab \qquad \text{---------(8)}$$

$$P_p.l_1\cos\delta = M_c + P_{PRC}l_3 \qquad \text{---------(9)}$$

After calculating the passive pressure for each failure surface, the minimum passive pressure corresponding to cohesion (P_{PC})is obtained in the same manner as in case (a).

Case (c) : $q \neq 0$, $\gamma = c = 0$

The equilibrium equation is given as:

$$P_p l_1 Cos\delta = Q.af + P_{PR}.l_3 \qquad \text{---------(10)}$$

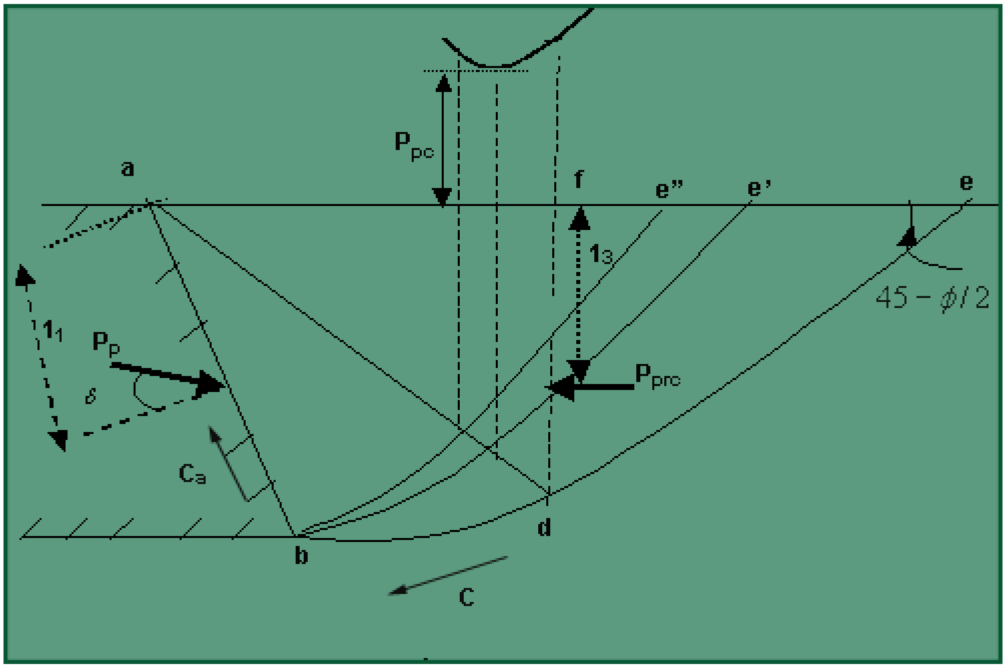


Fig. 2.21 Logarithmic Spiral Method for Determining Passive Earth Pressure

Case (a): $c \neq 0, \gamma = q = 0$

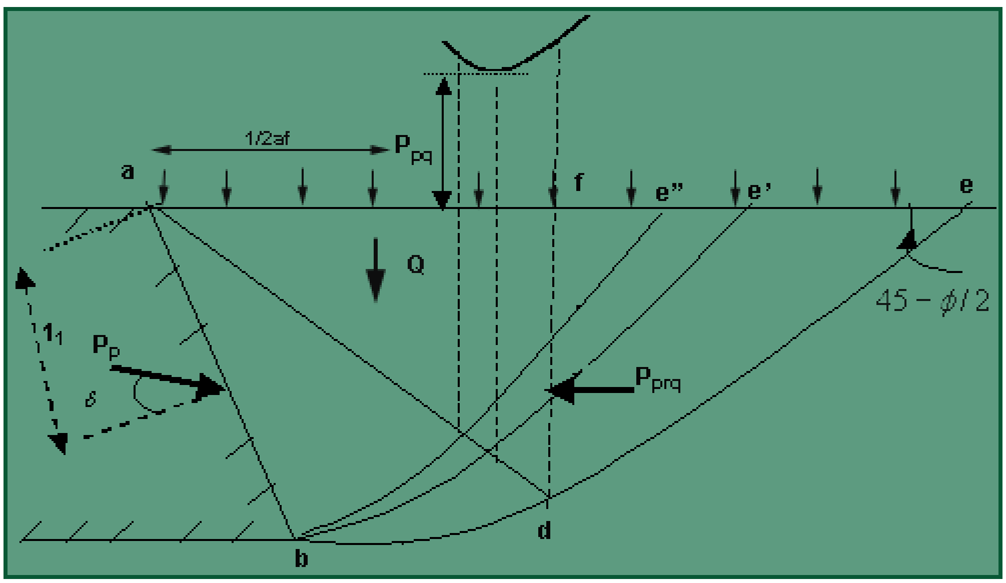


Fig. 2.22 Logarithmic Spiral Method for Determining Passive Earth Pressure

Case(c): $q \neq 0, c = \gamma = 0$

After getting the minimum values of passive pressure for the three cases, the absolute minimum is found out by summation of the three pressures. But the actual minimum pressure (P_{min}) obtained by superimposing each value of the three minimum passive pressures. Hence a single failure surface is not obtained for (P_{min}) (figure 2.23)

Absolute minimum pressure $P_{abs} = P_{PC} + P_{p\gamma} + P_{PQ}$

Error in using the method of superposition= $\dfrac{P_{min} - P_{abs}}{P_{min}} \times 100\% \approx \text{maximum } 30\%$

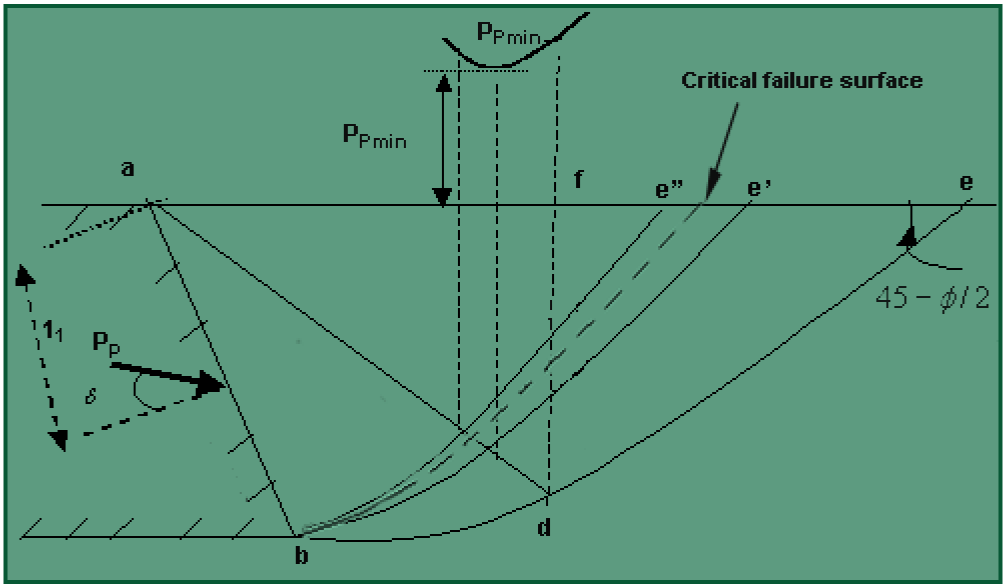


Fig. 2.23 Calculation of Minimum Passive Pressure.

Negative wall friction

In this case the passive pressure makes a negative angle of wall friction with the normal to the surface. The soil moves downwards with respect to wall. For negative wall friction only the log spiral failure surface is considered since the aim is to get minimum passive pressure. Kumar and Subbha Rao (1997) had done analysis of passive pressure under static condition using a composite failure surface for the negative friction angle case.

Seismic passive resistance in soil for negative wall friction

Choudhury and Subba Rao (2002) have analysed the seismic passive resistance for negative wall friction. In this analysis it was assumed that the rigid retaining structure is supporting dry, homogenous backfill with surcharge and that the occurrence of an earthquake does not affect the basic soil parameters. Uniform seismic accelerations were assumed to be acting at a particular time in both horizontal and vertical directions. The *limit equilibrium* method is used in the analysis of the earth pressure coefficients in soils for negative wall friction in the presence of pseudo-static seismic forces.

A retaining wall AB of vertical height H, wall batter angle α,ground slope β, wall friction angle δ,soil friction angle ϕ, coefficient of seismic horizontal acceleration K_h, and coefficient of seismic vertical acceleration K_v, was considered in the analysis (fig 2.24). The seismic passive resistance P_{pd} is split into three components as (i) unit weight component $c=q=0$, $\gamma \neq 0$ (ii) cohesion component $c \neq 0$, $\gamma = q = 0$ (iii) surcharge component $q \neq 0$, $\gamma = c = 0$.

The seismic passive resistance $P_{pd} = P_{pqd} + P_{pcd} + P_{p\gamma d}$ ---------(11)

Unlike the case of positive wall friction, focus point of the log spiral lies at the point F. The focus point is a variable and it is determined by varying η , where η is the inclination of the final radius r_f of the log spiral with the horizontal, so as to result in the minimum seismic passive resistance. The exit angle ξ at the point J on the ground is given by the following equation:

$$\xi = \frac{\pi}{4} - \frac{\phi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{K_h}{1-K_v}\right) + \frac{\beta}{2} + \frac{1}{2}\sin^{-1}\left\{\frac{\sin\left[\tan^{-1}\left(\frac{K_h}{1-K_v}\right) + \beta\right]}{\sin\phi}\right\} \quad \text{---------(12)}$$

for $K_h = K_v = 0$,eq(12) reduces to the same as Rankine's exit angle.

The forces considered are the seismic passive force P_{pd} acting on AB; the uniform surcharge pressure of q acting on AJ ;the weight W of the soil mass ABJ; the cohesive force C on the failure surface BJ;the normal force N on the failure surface BJ; the adhesive force C_a on the retaining wall-soil interface AB;the pseudo static force due to the seismic weight component for zone ABJ as Wk_h and Wk_v in the horizontal and vertical directions, respectively; and the pseudo-static forces due to Qk_h and Qk_v in the horizontal and vertical directions, respectively,on AJ.

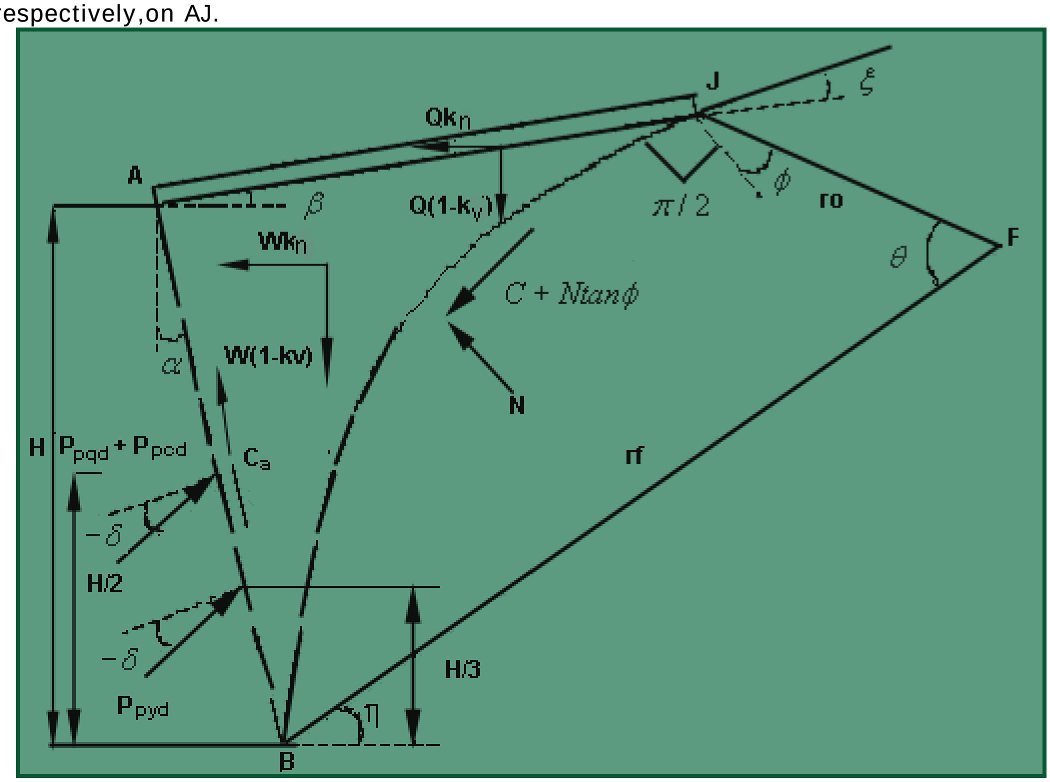


Fig. 2.24 Logarithmic Failure Surface and the Forces Considered

Total seismic passive pressure P_{pd} is given by:

$$P_{pd} = \left(2cHK_{pcd} + qHK_{pqd} + \frac{1}{2}\gamma H^2 K_{p\gamma d}\right)\frac{1}{\cos\delta} \qquad \text{----------(13)}$$

where $K_{P\gamma d}, K_{pcd}, K_{pqd}$ are the seismic passive earth pressures coefficients corresponding to the individual values of $P_{p\gamma d}$, P_{pcd} and P_{pqd} respectively.

The values of $K_{P\gamma d}, K_{pcd}, K_{pqd}$ are given by:

$$K_{P\gamma d} = \frac{2P_{p\gamma d}\cos\delta}{\gamma H^2} \qquad \text{----------(14)}$$

$$K_{pcd} = \frac{P_{pcd}\cos\delta}{2cH} \qquad \text{----------(15)}$$

$$K_{pqd} = \frac{P_{pqd}\cos\delta}{qH} \text{-} \qquad \text{----------(16)}$$

The analysis showed that the seismic passive pressure coefficients always decrease with the increase in the vertical seismic acceleration whereas, the horizontal seismic acceleration results in either an increase or decrease in the passive earth pressure coefficients, depending on the combinations of α, β and $\frac{\delta}{\phi}$.

Recap

In this section you have learnt the following

- Friction Circle Method
- Terzaghi's Analysis
- Log-Spiral Theory

Congratulations, you have finished Lecture 7. To view the next lecture select it from the left hand side menu of the page

Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 10 : Point of Application of Passive Earth Pressure [Section10.1 Rankine's & Coulomb's Theory]

Objectives

In this section you will learn the following

- Rankine’s theory
- Coulomb’s theory
- Method of horizontal slices given by Wang (2000)
- Distribution of the earth pressure
- Height of application of the resultant earth pressure
- Earth pressure distribution in seismic case given by Choudhury et al. (2002)
- Expression for the seismic passive earth pressure

10 POINT OF APPLICATION OF PASSIVE EARTH PRESSURE

1 Rankine's theory

In Rankine's earth pressure theory the intensity of earth pressure at each depth is known. So, point of application of the passive earth pressure is known at any depth as shown in fig. 2.36.

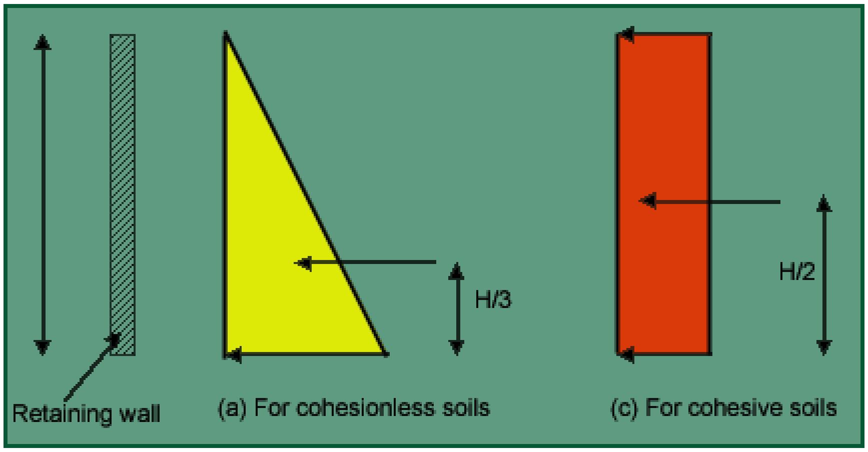


Fig. 2.36 Point of application of passive earth pressure (Rankine's theory)

2 Coulomb's theory

In the coulomb's method only the total earth pressure value acting on the retaining structure can be calculated. The point of application of the earth pressure can be calculated from coulomb's assumption that all points on the back of the retaining wall are essentially considered as feet of the failure surface. So, for each failure surfaces as shown in fig. 2.37, earth pressure intensity can be calculated. Let the earth pressure intensity for failure surface 1-1' is p a at depth z from top of retaining wall and the earth pressure intensity for failure surface 2-2' is p a' at depth (z +dz) from top of retaining wall.

Then, $dp_a = p_a' - p_a$,

Therefore, (dp_a / dz) can be estimated. This gives the variation of the earth pressure intensity along the depth. As the variation of earth pressure is known, earth pressure diagram can be plotted and point of application of the resultant earth pressure can be found.

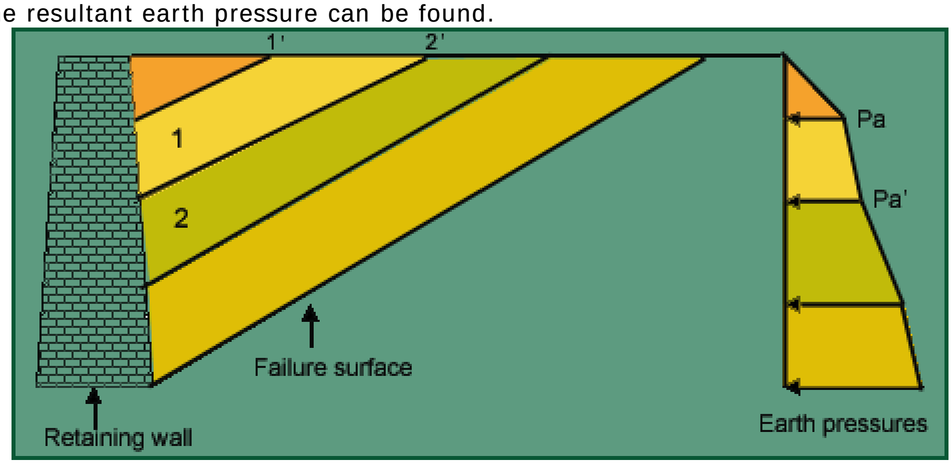


Fig. 2.37Point of application of passive earth pressure (Coulomb's theory)

The assumption made by Coulomb is justified in case of retaining walls, because no retaining wall can fail without yielding in a manner that satisfies the deformation condition for the plastic state. Coulomb, however, did not satisfy this deformation condition. As a consequence, the theory was commonly used for computing the earth pressures against the lateral supports that did not satisfy the deformation condition, such as bracing in open cut.

Method of horizontal slices given by Wang (2000)

The approximate distribution of the earth pressure can be determined numerically by computing resultant earth pressure at various depths along the wall. The unit earth pressure calculated on the basis of Coulomb's and Rankine's theories varies linearly with depth. But, from experimental data it is found that the unit earth pressure is curvilinearly distributed on the back of the retaining wall. Wang (2000) has considered the Coulomb's concept as the basis for determining the intensity of earth pressure by the method of horizontal slices. A sliding wedge considered for the analysis is shown in fig.2.38 (a). An element of thickness dy is considered from the wedge at depth y below the surface. The forces on the element are shown in fig.2.38 (b).

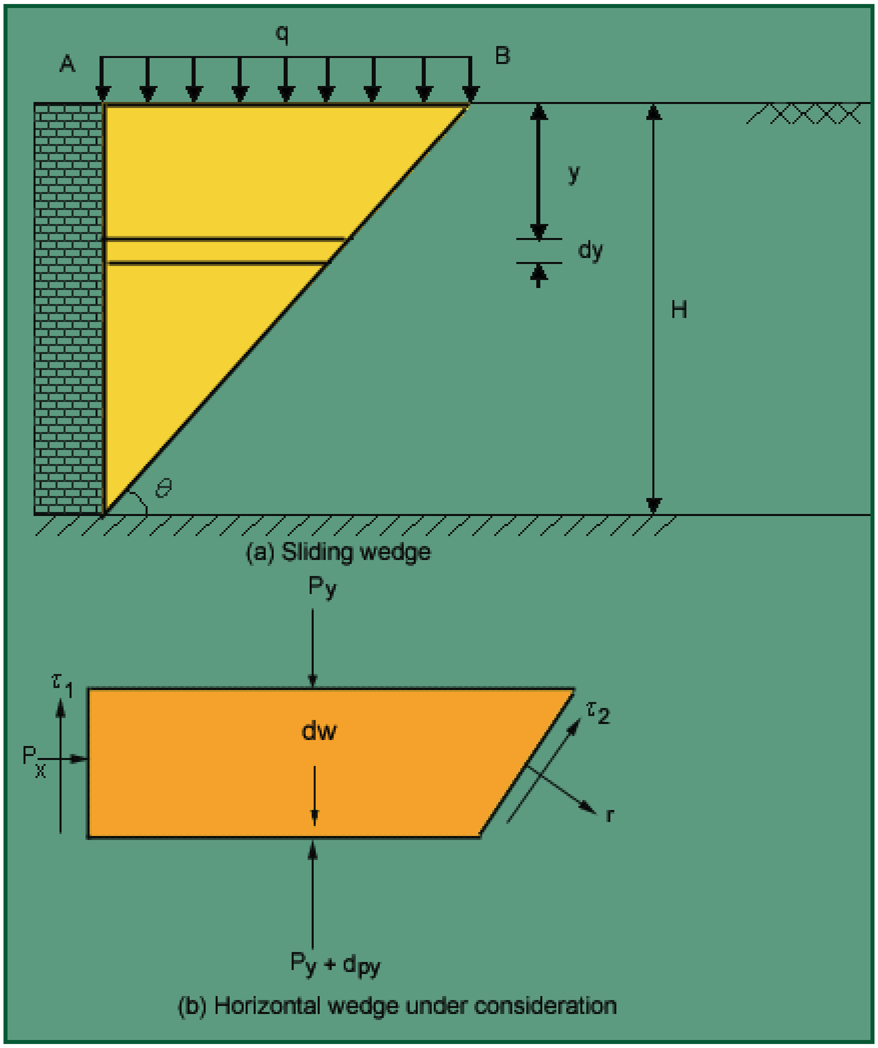


Fig. 2.38 Analytical model

Where,

p_y is the vertical pressure on the top of the element,

(p_y +d p_y) is the vertical reaction on the bottom of the element,

p_x is the horizontal reaction on the retaining wall,

τ_1 is shear between the back fill and the back of retaining wall,

τ_2 is shear between the sliding backfill and the remaining backfill at rest,

dw is the weight of the element.

The shearing forces on the top and bottom of the element are neglected, considering that the wedge slides as a whole. Considering the equilibrium condition of the vertical forces on the element, the basic equation for the unit earth pressure on the retaining wall is given by,

$$\frac{dp_y}{dy} = \left[1 - \frac{\cos(\theta - \phi - \delta)\tan\theta}{\sin(\theta - \phi)\cos\delta}\right]\frac{p_y}{H - y} + \gamma$$

where,

δ is the friction angle between the back of retaining wall and the backfill,

ϕ is the internal friction angle of the backfill,

H is the height of the retaining wall,

γ is density of the backfill material.

Then,

$$p_y = \left(q - \frac{\gamma H}{aK-2}\right)\left(\frac{H-y}{H}\right)^{aK-1} + \frac{\gamma H}{aK-2}\frac{H-y}{H}$$

where,

$$a = \left[1 - \frac{\cos(\theta-\phi-\delta)\tan\theta}{\sin(\theta-\phi)\cos\delta}\right],$$

K is the lateral pressure coefficient.

$$p_x = K\left\{\left(q - \frac{\gamma H}{aK-2}\right)\left(\frac{H-y}{H}\right)^{aK-1} + \frac{\gamma H}{aK-2}\frac{H-y}{H}\right\}$$

Resultant earth pressure on the wall

Total horizontal earth pressure can be given by,

$$P_x = \int_0^H p_x dy = \frac{\sin(\theta-\phi)\cos\delta\cos\theta}{\cos(\theta-\phi-\delta)}\left(qH + \frac{1}{2}\gamma H^2\right)$$

The total shearing force on the wall can be given as,

$$T_1 = \int_0^H \tau_1 dy = \frac{\sin(\theta-\phi)\cos\delta\cos\theta}{\cos(\theta-\phi-\delta)}\left(qH + \frac{1}{2}\gamma H^2\right)$$

The resultant force on the wall is as,

$$P = \frac{\sin(\theta-\phi)\cot\theta}{\cos(\theta-\phi-\delta)}\left(qH + \frac{1}{2}\gamma H^2\right)$$

If the surcharge q =0, then,

$$P = \frac{\sin(\theta-\phi)\cot\theta}{\cos(\theta-\phi-\delta)}\left(\frac{1}{2}\gamma H^2\right)$$

The above equation is same as given by Coulomb's theory.

Distribution of the earth pressure

The unit earth pressure is given as,

$$p = \frac{K}{\cos\delta}\left\{\left(q - \frac{\gamma H}{aK-2}\right)\left(\frac{H-y}{H}\right)^{aK-1} + \frac{\gamma H}{aK-2}\frac{H-y}{H}\right\}$$

This is curvilinearly distributed and related to the lateral pressure coefficient K.

When K=1/a, then,

$$p = (q - \gamma y)\frac{1}{a\cos\delta}$$

This represents linearly distributed earth pressure.

Height of application of the resultant earth pressure

In case where the earth pressure is linearly distributed, the height of application of the resultant earth pressure (H_p) from the wall bottom is,

$$H_p = \frac{1}{3} H \frac{3q - \gamma H}{2q - \gamma H}$$

and when q=0, then H_p = H/3.

For the curvilinearly distributed earth pressure,

$$H_p = \frac{M}{p_x} = \left[\frac{1}{3} + \frac{aK - 1}{3(aK + 1)}\right] H \frac{3q + \gamma H}{2q + \gamma H}$$

where,

M is the resultant moment of the earth pressure about the wall bottom

$$= \int_0^H (H - y) p_x dy = \frac{KH^2}{aK + 1}\left(q + \frac{1}{3}\gamma H\right)$$

Earth pressure distribution in seismic case given by Choudhury et al. (2002)

A method of horizontal slices has been suggested for obtaining seismic passive earth pressure distribution by considering seismic forces in a pseudo-static manner. Only planar rupture surfaces have been considered and hence wall friction angle has been restricted upto one-third the soil friction angle. This approach results in the same seismic passive earth pressure coefficients as that obtained by Mononobe-Okabe approach, besides giving additional information about the distribution of earth pressures. It has been found that in the seismic case, passive resistance acts at a point other than at 1/3rd from the base of the wall. Under seismic conditions, the extension of failure zone is more than that under static conditions.

This method is an extension to Wang's approach and is suggested for determining the seismic passive earth pressure distribution for a rigid inclined retaining wall supporting cohesionless backfill.

Analytical Model

The method of horizontal slices is considered in the analysis. In Fig.2.39a, a rigid retaining wall of vertical height H, supporting dry, homogeneous cohesionless backfill material with horizontal ground is shown. The seismic forces are considered as pseudo-static forces along with other static forces. The equilibrium of each elemental slice is considered. It is assumed that the occurrence of earthquake does not affect the basic soil parameters: soil friction angle f and soil unit weight γ .

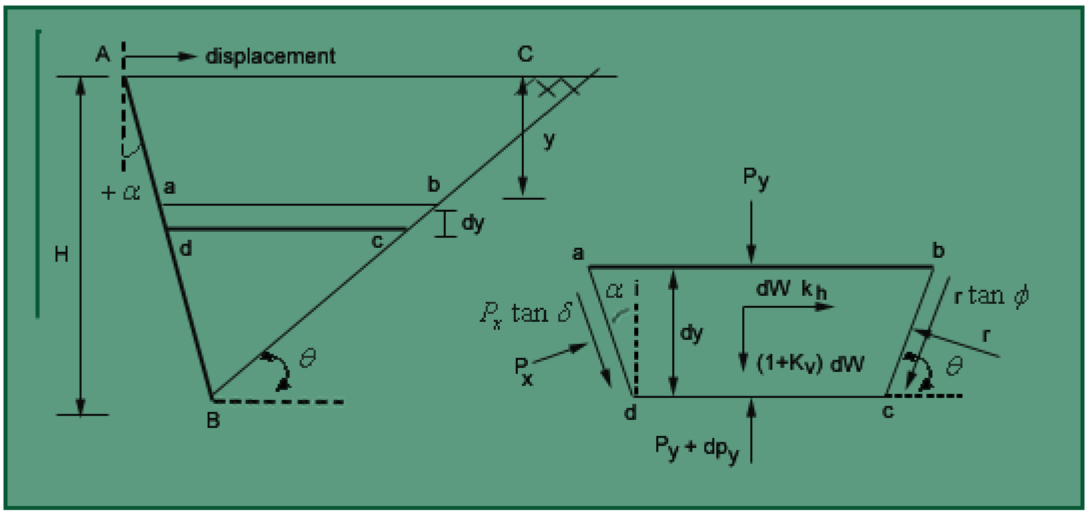


Fig. 2.39 (a)Analytical model (b) Free body diagram

Uniform seismic accelerations $K_h g$ and $K_v g$ are assumed in the horizontal and vertical directions respectively in the domain under consideration. As a first step, only planar rupture surfaces have been considered and to keep this assumption valid, wall friction angle δ has been restricted to less than or equal to $\phi/3$ as shown by Terzaghi (1943). In Fig.2.39b, the free body diagram of an elemental slice shows the action of different forces. The thickness of the slice is dy, at a depth of y from the top ground surface. The vertical pressure p_y is acting on the top of the element and (p_y + d p_y) on the bottom of the element. The reaction p_x normal to the wall and the shear force p_x tan δ are acting on the interface between the retaining wall and the backfill material. The normal force r and the shear force for tan ϕ act on the sliding surface. The other forces are, the weight dW of the element, the seismic forces dWK_h in the horizontal direction and dWK_v in the the vertical direction. The critical directions of these seismic forces are as shown in Fig.4.9b. The horizontal slip planes are assumed as principal planes.

Unit earth pressure on the retaining wall

Considering the equilibrium condition of the vertical forces on the element, the basic equation for the unit earth pressure on the retaining wall is given by,

$$\frac{dp_y}{dy} = \frac{p_y}{H-y}(1+K) + \gamma m$$

Expression for the seismic passive earth pressure

Resolving all the forces in the vertical and horizontal directions, from the boundary condition p_y = 0 at y = 0, and ignoring higher order differential terms and upon simplification, the expression for the seismic passive earth pressure at any depth y is obtained as,

$$p_x = K\left[\frac{\gamma n}{2+aK}\left\{\frac{H^{(2+aK)}}{(H-y)^{(1+aK)}} - (H-y)\right\}\right]$$

where p_x = seismic passive pressure at any depth y from top acting normal to the wall

K = lateral earth pressure coefficient

$n = (1 - K_v - K_x b)$

$b = \cot(\theta + \phi)$

$$a = \frac{(\tan\phi - \cot\theta)(1 + \tan\alpha \tan\delta)}{(\tan\alpha + \cot\theta)(1 + \tan\phi \cot\theta)} + \frac{(\tan\delta - \tan\alpha)}{(\tan\alpha + \cot\theta)}$$

d = wall friction angle

θ = angle of inclination of the failure plane with horizontal

Integrating over the height of the wall, the total passive resistance p_x is given by,

$$P_x = \int_0^H p_x dy = \frac{\gamma m H^2}{a(2+aK)}\left(1000^{aK} - 1\right) - \frac{K\gamma m H^2}{2(2+aK)}$$

The value of coefficient K has been obtained from the static analysis by comparing the results with that of Coulomb. The equivalent seismic passive earth pressure coefficient with respect to normal to the wall is found out as,

$$K_{pd} = \frac{2P_x}{\gamma H^2}$$

The critical value of θ is obtained by minimizing p_x with respect to θ keeping all other parameters constant, and it is found to exactly match with the Coulombic values for static case.

Height of application of the resultant earth pressure

M is the resultant moment of the earth pressure about the wall bottom

$$= \int_0^H (H-y)p_x dy = \frac{\gamma m K H^2}{aK+2}\left(\frac{1}{(aK-1)}\left(1000^{(aK-1)} - 1\right) - \frac{1}{3}\right)$$

The point of application of the passive earth pressure from base $= h = \dfrac{M}{p_x}$

Recap

In this section you have learnt the following

- Rankine’s theory
- Coulomb’s theory
- Method of horizontal slices given by Wang (2000)
- Distribution of the earth pressure
- Height of application of the resultant earth pressure
- Earth pressure distribution in seismic case given by Choudhury et al. (2002)
- Expression for the seismic passive earth pressure

Congratulations, you have finished Lecture 10. To view the next lecture select it from the left hand side menu of the page